Journal of Public Economics 93 (2009) 867–878

Contents lists available at ScienceDirect

Journal of Public Economics

journal homepage: www.elsevier.com/locate/jpube

Labor supply elasticity and social security reform ☆

Selahattin İmrohoroğlu *, Sagiri Kitao

University of Southern California, Marshall School of Business, Finance and Business Economics Department, United States

ARTICLE INFO

Article history:
Received 24 August 2008
Received in revised form 9 May 2009
Accepted 12 May 2009
Available online 22 May 2009

JEL classification:
E2
E6
H55
J2

Keywords:
Social security reform
Labor supply elasticity

ABSTRACT

Previous literature on social security reform has used a variety of period utility functions and calibrated values for the intertemporal elasticity of substitution (IES) in labor. In this paper, we show that the effects of social security reforms on aggregate labor supply are invariant to plausible values of the IES, but the effect of such reforms on the profile of hours over the life-cycle is highly sensitive to the IES. We first establish these results analytically in a simple partial-equilibrium setting and then demonstrate their robustness in a general equilibrium model calibrated to match key U.S. macroeconomic indicators. We find that the aggregate effects are similar regardless of the wide range of the values of IES used in calibrated economies. However, social security reform leads to a large reallocation of hours worked over the life-cycle, from early years to later working years, and the size of this reallocation significantly increases with the IES.

1. Introduction

There is a large literature on social security reform that uses different utility functions with different estimates of the intertemporal elasticity of substitution in labor supply (IES).[1] In this paper, we investigate the implications of making different choices about the period utility function and values of the IES for the policy debate on social security. We show that the effects of social security reforms on aggregate labor supply are invariant to plausible values of the IES, but the effect of such reforms on the profile of hours over the life-cycle is highly sensitive to the IES. We first establish these results analytically in a simple partial-equilibrium setting and then demonstrate their robustness in a general equilibrium model calibrated to match key U.S. macroeconomic indicators.

We start with a simplified theoretical framework. We consider a two-period, deterministic, partial-equilibrium overlapping generations model and a period utility function which is separable in consumption and leisure. Building on Chetty (2006) we show analytically that the overall impact on labor supply of a permanent increase in wage depends on the relative strengths of the substitution and income effects. In the special case of logarithmic utility over consumption, these effects cancel out and there is no effect on labor supply. Furthermore the curvature of the disutility of work has no effect either. In the more general case, when the coefficient of relative risk aversion is less (more) than unity, the substitution (income) effect dominates and the labor supply will rise (fall) with an increase in wage. The magnitude of the response will be greater if the IES is larger. In addition, a permanent decrease in the interest rate will lower the growth rate of consumption which flattens the age-hours profile and individuals supply more hours at older ages. These analytical results provide the economic intuition behind the effects of price changes (induced, for example, by social security reform) on labor supply. However, extending these analytical results to non-separable utility functions or to a general equilibrium framework proved difficult. In particular, in a general equilibrium setting, social security reform raises the capital-labor ratio and causes both an increase in the wage rate and a decrease in the interest rate. In this case, even with a separable utility function, the overall effects of reform on both the aggregate labor supply and its allocation over the life-cycle become ambiguous.

Once we demonstrate the results and intuition with the analytical framework, the next task of the paper is to conduct an extensive quantitative analysis using a general equilibrium model populated by long-lived individuals facing mortality and idiosyncratic income risks and borrowing constraints. Individuals in this economy choose consumption and hours worked until a mandatory retirement age. The benefit and taxation rules of social security are implemented according to the formulas used by the Social Security Administration (SSA). The fiscal authority taxes capital and labor income and consumption to finance an exogenous quantity of government

☆ We would like to thank the editor, two anonymous referees, and participants of seminars at GRIPS in Tokyo and the Federal Reserve Bank of San Francisco.

* Corresponding author.
E-mail addresses: selo@marshall.usc.edu (S. İmrohoroğlu), skitao@marshall.usc.edu (S. Kitao).

[1] For example, see Feldstein (1985), Hubbard and Judd (1987), İmrohoroğlu et al. (1995), İmrohoroğlu et al. (2003), Cooley and Soares (1999), Boldrin and Rustichini (2000), Casamatta et al. (2002) Huang et al. (1997), De Nardi et al. (1999), Kotlikoff et al. (1999), Conesa and Krueger (1999), Fuster et al. (2007) and Nishiyama and Smetters (2007).

0047-2727/$ – see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.jpubeco.2009.05.002

purchases and interest payments on its debt. We calibrate the model by using both micro data and aggregate data on the U.S. economy. We then conduct counterfactual (revenue-neutral) experiments by reforming social security, either by downsizing the system by 50%, or by totally eliminating it. Our goal is to understand the long-run effect of social security reforms and we focus on how the strength of IES affects the properties of the steady state under alternative policies. Our analysis is based on the comparison of the two steady states, one implied by the current pay-as-you-go social security system and the other by a reform towards a fully-funded system.

Our main quantitative finding is that social security reform leads to similar aggregate outcomes for a wide range of the IES. For example, when we take the IES as 0.1, half privatization leads to an increase of 10.8% in the capital stock, an increase of 0.03% in the average work hours and a decrease of 5.8% in the budget-clearing labor income tax rate, whereas the changes are 9.8%, 0.11% and −5.9%, respectively, when the IES is taken as 1.0, a ten-fold increase in the intertemporal responsiveness of labor. Long-run welfare is also similar: individuals strongly prefer to be born into the reformed steady state; they are willing to give up 5.0% and 5.4% consumption, respectively, in the unfunded steady state in order to be born in the reformed one. However, these similarities at the aggregate level hide significant differences in the allocation of work hours over the life-cycle. With reform (half privatization) individuals shift work from the early years in the life-cycle to later years, regardless of the IES. However, with an IES equal to unity, this reallocation is quantitatively much more significant than that in the case of an IES of 0.1. This reallocation is even larger in the case of full privatization. Therefore, the IES used in a study of social security reform leaves the aggregate implications unchanged to a large extent in the long run, but matters significantly when analyzing the life-cycle implications of the reform.[2]

We would like to emphasize that this paper explores the sensitivity of gains from social security reform to changes in the IES, but should not be interpreted as an exercise attempting to evaluate optimal social security reform. The overall welfare consequences of social security reform depend upon transitional costs and a variety of other parameters and factors beyond the IES.

The remainder of the paper is organized as follows. We present a simple two-period model and its analytical results in Section 2. The large-scale, general equilibrium model is described in Section 3. The calibration details are given in Section 4. Section 5 presents our numerical findings. Section 6 conducts a sensitivity analysis and concluding remarks are given in Section 7.

2. Simple partial-equilibrium models and intuition

2.1. A general multi-period model

Consider an economy with a complete market, populated by J overlapping generations. Households derive utility from consumption, incur disutility from labor and maximize their life-time utility by optimally choosing the sequence of consumption, saving and labor supply over the life-cycle.

We start with preferences that are separable in consumption and labor supply and over time:

$$\sum_{j=1}^{J} \beta^{j-1}\left[u\left(c_j\right) - v\left(\ell_j\right)\right],$$

where c_j and ℓ_j denote consumption and labor supply at age j, respectively. $u(c)$ represents the utility from consumption with $u'>0$ and $u''<0$, $v(\ell)$ disutility from work with $v'>0$ and $v''>0$. β is the subjective discount factor. The maximization is subject to the life-time budget constraint:

$$\sum_{j=1}^{J}\left(\frac{1}{R}\right)^{j-1} c_j = \sum_{j=1}^{J}\left(\frac{1}{R}\right)^{j-1} w\ell_j, \tag{1}$$

where w and R are the wage and gross interest rate that are exogenously given.

First-order conditions with respect to the labor supply and consumption are given as

$$u'\left(c_j\right)w = v'\left(\ell_j\right), \tag{2}$$

$$u'\left(c_j\right) = \beta R u'\left(c_{j+1}\right). \tag{3}$$

From Eqs. (2) and (3) we can make the following general points that extend the intuition in Chetty (2006):

- When there is a permanent increase in the wage w, the effect on labor supply depends on the magnitude of the decline in $u'(c_j)$ (substitution effect) relative to the increase in the wage rate (income effect) in Eq. (2).
  A one-percent increase in the wage will raise the consumption at any given level of labor supply by 1%, which lowers the marginal utility of consumption by $\varepsilon_c = -(\partial u_c/\partial c)(c/u_c)$, that is, the coefficient of relative risk aversion (CRRA).
  - In the case of log utility, $\varepsilon_c = 1$ and the substitution and income effects exactly offset each other, resulting in no change in labor supply. Obviously, the curvature of $v(\ell)$ (IES) has no effect on aggregate labor supply in this case.
  - If $\varepsilon_c < 1$ (>1), the *substitution effect (the income effect)* dominates and the labor supply will rise (fall) upon an increase in the wage. The magnitude of the response will be larger if the labor supply elasticity is higher.
- A permanent change in the interest rate will affect the labor supply profile through the change in the marginal rate of substitution (or the optimal growth rate of consumption). Using Eqs. (2) and (3), we obtain

$$\frac{v'\left(\ell_j\right)}{v'\left(\ell_{j+1}\right)} = \frac{u'\left(c_j\right)}{u'\left(c_{j+1}\right)} = \beta R.$$

  - A decrease in the interest rate, for example, will lower (increase) the growth rate of consumption (labor supply) and households extend more work effort at older ages.

In the next section, we assume a particular functional form for the utility from consumption and work disutility to highlight the roles of the CRRA and IES.

2.2. A two-period model with analytical solutions

Households live for two periods. They choose sequences of consumption $\{c_j\}_{j=1}^{2}$ and labor supply $\{\ell_j\}_{j=1}^{2}$ in order to maximize life-time utility. Agents can save or borrow at the gross interest rate R. Both the interest rate and the wage per efficiency unit w are exogenously given. Labor income is given as $w\varepsilon_j\ell_j$, where ε_j is

[2] Rogerson and Wallenius (forthcoming) find large macro elasticities, 2.25 to 3, when they vary the IES from 0.05 to 1.25 in an experiment in which the labor income tax rate is raised from 30% to 50% with the proceeds returned in a lump sum fashion. They use a continuous time life cycle model of a complete market with no borrowing constraint in which individuals choose not only the hours worked but also the fraction of the life cycle spent in market activities. The decrease in aggregate hours worked is essentially the same for any IES in the range they consider. However, the hours profile is affected more significantly.

age-dependent labor productivity. The household's problem is defined as follows.

$$\max_{\{c_1,c_2,\ell_1,\ell_2\}} \frac{c_1^{1-\sigma}}{1-\sigma}+\beta\frac{c_2^{1-\sigma}}{1-\sigma}-\frac{\ell_1^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}-\beta\frac{\ell_2^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}},$$

subject to

$$c_1+\frac{c_2}{R}=w\varepsilon_1\ell_1+\frac{w\varepsilon_2\ell_2}{R}. \tag{4}$$

The parameters σ and γ represent the CRRA and IES respectively. The first-order conditions are given as

$$\frac{c_2}{c_1}=(\beta R)^{\frac{1}{\sigma}}, \tag{5}$$

$$\ell_j^{\frac{1}{\gamma}}=c_j^{-\sigma}w\varepsilon_j \text{ for } j=1,2. \tag{6}$$

Using Eqs. (5) and (6) in Eq. (4), we obtain

$$c_1=\left\{w^{1+\gamma}\left[\varepsilon_1^{1+\gamma}+\frac{\varepsilon_2^{1+\gamma}}{\beta^\gamma R^{1+\gamma}}\right]\left(1+R^{\frac{1-\sigma}{\sigma}}\beta^{\frac{1}{\sigma}}\right)^{-1}\right\}^{\frac{1}{1+\sigma\gamma}}. \tag{7}$$

Using Eq. (7) in Eqs. (5) and (6), we obtain

$$\ell_1=c_1^{-\sigma\gamma}(w\varepsilon_1)^\gamma=w^{\frac{\gamma(1-\sigma)}{1+\sigma\gamma}}A_1,$$

where

$$A_1\equiv\left\{\left[\varepsilon_1^{1+\gamma}+\frac{\varepsilon_2^{1+\gamma}}{\beta^\gamma R^{1+\gamma}}\right]\left(1+R^{\frac{1-\sigma}{\sigma}}\beta^{\frac{1}{\sigma}}\right)^{-1}\right\}^{-\frac{\sigma\gamma}{1+\sigma\gamma}}\varepsilon_1^\gamma, \text{ and,}$$

$$\ell_2=c_1^{-\sigma\gamma}\left[\frac{w\varepsilon_2}{\beta R}\right]^\gamma=w^{\frac{\gamma(1-\sigma)}{1+\sigma\gamma}}A_2, \tag{8}$$

where

$$A_2\equiv\left\{\left[\varepsilon_1^{1+\gamma}+\frac{\varepsilon_2^{1+\gamma}}{\beta^\gamma R^{1+\gamma}}\right]\left(1+R^{\frac{1-\sigma}{\sigma}}\beta^{\frac{1}{\sigma}}\right)^{-1}\right\}^{-\frac{\sigma\gamma}{1+\sigma\gamma}}\left(\frac{\varepsilon_2}{\beta R}\right)^\gamma.$$

First, consider the effect of a permanent change in the wage rate on labor supply ℓ_j ($j=1,2$):

$$\frac{\partial\ell_j}{\partial w}=\frac{\gamma(1-\sigma)}{1+\sigma\gamma}w^{\frac{\gamma(1-\sigma)}{1+\sigma\gamma}-1}A_j\begin{cases}>0 \text{ if } \sigma<1,\\ =0 \text{ if } \sigma=1, \text{ (log in consumption)}\\ <0 \text{ if } \sigma>1.\end{cases}$$

With $\sigma<1$, the substitution effect dominates the income effect and a rise in the wage rate will induce more work in both periods. In the case of $\sigma=1$ (log utility in consumption), the substitution and income effects exactly offset each other and there will be no effect on labor supply.

Now consider the effect of a change in the interest rate on the labor supply. From the first-order condition (6), one can see that labor supply will fall when the optimal consumption rises. It can be shown that the derivative of c_1 with respect to the interest rate R is unambiguously negative when the coefficient of relative risk aversion does not exceed unity($\sigma\leq 1$) but the sign is ambiguous otherwise. Under our time-separable utility specification, low relative risk aversion implies a high intertemporal elasticity of substitution and a rise in the interest rate will induce agents to reduce consumption in the first period in order to save more and exploit the opportunity of the higher interest rate. Labor supply in such a case is higher in the first period. The change in the labor supply in the second period is ambiguous for any value of the coefficient of relative risk aversion. An income effect may dominate, in which case the aggregate labor supply rises. See Appendix A for the detailed algebra.

In order to see the effects on the intertemporal allocation of labor supply, we note that the growth rate of the labor supply g_ℓ is given as

$$g_\ell=\frac{\ell_2}{\ell_1}=\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^\gamma(\beta R)^{-\gamma}.$$

The steeper the productivity profile $\varepsilon_2/\varepsilon_1$ is and the lower the interest rate and/or the discount rate are, the more work effort agents will allocate when they are older in period 2. The derivatives of the slope with respect to the wage and interest rate are given as

$$\frac{\partial g_\ell}{\partial w}=0,$$
$$\frac{\partial g_\ell}{\partial R}=-\gamma\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^\gamma(\beta R)^{-\gamma-1}\beta.$$

A change in the wage rate will have only a level effect on both ℓ_1 and ℓ_2, but the growth rate is not affected. When the interest rate falls, the growth rate rises, that is, agents work relatively more intensively when they are older and less so when they are younger. This effect comes from the change in the growth rate of consumption over the life-cycle and the income effect, in which agents consume relatively more when they are young.

It becomes more difficult to extend these analytical results to the case of non-separable utility functions or to a general equilibrium framework. For this reason, we introduce a quantitative general equilibrium model of overlapping generations in the next section.

3. A quantitative general equilibrium model

3.1. Demographics

In each period the economy is populated by overlapping generations of individuals of age $j=1,2,\ldots J$, who face lifespan uncertainty until the maximum possible age J. We denote the conditional probability of survival from age j to age $j+1$ with s_j, with $s_J=0$. The size of new cohort grows at a constant rate n. Accidental bequests are collected and distributed as a lump-sum transfer to the entire population. We restrict our attention to steady states and omit all time subscripts.

3.2. Endowments and preferences

Households enter the economy with no assets. They are endowed with one unit of time that can be used for leisure or market work. Households' earnings are given by $w\varepsilon_j\eta\ell$, where w is the market wage, ε_j is the deterministic age-specific productivity, η is an idiosyncratic labor productivity that evolves stochastically, and ℓ is the endogenously chosen hours of work.

Households order the sequences of consumption and labor supply over the life-cycle according to a time-separable utility function

$$E\left\{\sum_{j=1}^{J}\beta^{j-1}u\left(c_j,\ell_j\right)\right\},$$

where β is the subjective discount factor and the expectation is with respect to the shocks associated with the time of death and idiosyncratic labor productivity, and consumption and labor supply at age j are denoted by c_j and ℓ_j, respectively.

3.3. Technology

There is a representative firm that runs a constant returns to scale technology of the form $Y = F(A,K,L) = K^{\alpha}(AL)^{1-\alpha}$, where K and L are aggregate capital and labor inputs and α is capital's share of output. A is the total factor productivity which we assume is constant. Capital depreciates at a constant rate $\delta \in (0,1)$. The firm rents capital and hires labor from households in competitive markets, where factor prices r and w are equated to the marginal productivities.

3.4. Social security

In the benchmark economy, the government operates a pay-as-you-go pension system similar to the current U.S. system. Working households pay a proportional tax τ^{ss} on their labor income up to the maximum amount of y^{ss}, after which the social security tax rate is zero. Each retired agent receives a constant benefit ss, which is a concave function of an individual's average life-time earnings that captures the progressivity of the U.S. social security system. We will consider different reforms in the direction of a privatized system in Section 5.

3.5. Market structure

The markets are incomplete and households cannot insure against the idiosyncratic labor income and mortality risks by trading state-contingent assets. They can, however, hold one-period riskless assets to imperfectly self-insure against idiosyncratic risks. We assume that agents are not allowed to borrow against future income, i.e. $a_j \geq 0$ for all j.

3.6. Households problem

Households are heterogeneous in four dimensions summarized by a state vector $x = \{j,a,\eta,e\}$, where age is j, assets accumulated in the previous age are denoted by a, the idiosyncratic labor productivity is η, and e represents the cumulated labor earnings that determine the retirement benefit.

We compute the household's problem recursively. The value function $V(x)$ of an individual in state x is given by

$$V(j,a,\eta,e) = \max_{c,\ell,a'} \left\{ u(c,\ell) + \beta s_j E[V(j+1,a',\eta',e')] \right\}$$

subject to

$$\begin{aligned}
&c + a' = (1+r)(a+b) + w\varepsilon_j\eta\ell + ss(x) - \Upsilon(x),\\
&a' \geq 0,\\
&e' = \left[(j-1)e + w\varepsilon_j\eta\ell\right] / j, \text{ for } j < j_R,\\
&e' = e, \text{ for } j \geq j_R,\\
&\Upsilon(x) = \tau^{\ell} w\varepsilon_j\eta\ell + \tau^{ss}\min\left\{w\varepsilon_j\eta\ell, y^{ss}\right\} + \tau^a r(a+b) + \tau^c c
\end{aligned}$$

where $\Upsilon(x)$ denotes the taxes paid by a household in state x. The cumulated labor earnings e evolves according to the sequence of the realization of labor productivity shocks and endogenously chosen hours of work profile.

3.7. Fiscal policy

Besides the social security tax, the government raises revenue from taxes on labor income, capital income and consumption at proportional rates denoted by τ^{ℓ}, τ^a and τ^c, and issues one-period riskless debt D'. The government debt and tax revenue finance the payment of pensions for the retired, an exogenously given level of public purchases of goods and services G and the servicing and repayment of the debt. The labor income tax rate τ^{ℓ} is set so that the following consolidated government budget constraint is satisfied every period.

$$\begin{aligned}
G + (1+r)D + \sum_x ss(x)\mu(x) = \sum_x [\tau^{\ell} w\varepsilon_j\eta\ell(x) + \tau^{ss}\min\left\{w\varepsilon_j\eta\ell(x), y^{ss}\right\}\\
+ \tau^a r(a(x)+b) + \tau^c c(x)]\mu(x) + D',
\end{aligned} \tag{9}$$

where $\mu(x)$ denotes the measure of individuals in state x, D is the debt issued in the previous period and D' is the proceeds of the debt issued in the current period.

3.8. Equilibrium

For a given set of exogenous demographic parameters $\{s_j\}_{j=1}^{J}$ and $\{n\}$ and government policy variables $\{G,D',ss,\tau^{ss},\tau^a,\tau^c\}$, a stationary competitive equilibrium consists of households' decision rules $\{c(x), \ell(x), a(x)\}$ for each state x, factor prices $\{w,r\}$, labor income tax rate $\{\tau^{\ell}\}$, a lump-sum transfer of accidental bequests $\{b\}$ and the measure of individuals $\{\mu(x)\}$ that satisfy the following conditions:

1. Households' allocation rules solve their recursive optimization problems defined in Section 3.6.
2. Factor prices are determined competitively, i.e. $w = F_L(A,K,L)$ and $r = F_K(A,K,L) - \delta$.
3. The lump-sum bequest transfer is equal to the amount of assets left by the deceased.

$$b = \sum_x a(x)\left(1 - s_{j-1}\right)\mu(x). \tag{10}$$

4. The labor and capital markets clear.

$$L = \sum_x \varepsilon_j\eta\ell(x)\mu(x), \tag{11}$$

$$K = \sum_x (a(x)+b)\mu(x) - D. \tag{12}$$

5. The labor income tax satisfies the government budget constraint defined in Eq. (9).
6. The goods market clears.

$$\sum_x c(x)\mu(x) + K' + G = Y + (1-\delta)K. \tag{13}$$

4. Calibration

4.1. Demographics

One model period corresponds to a year. We assume that households enter the economy at age 20 ($j = 1$), retire from work at age 65 ($j_R = 46$) and live up to the maximum age of 100 ($J = 81$). We use the study of Bell and Miller (2005) for the current age-dependent conditional survival probabilities in the U.S. We set the growth rate n of the new entrants to the economy to 1.1%, the long-run average in the U.S. The survival probabilities and the population growth rate imply the old dependency ratio of 24.5%, defined as the ratio of the population aged 65 and over to that between 20 and 64.

In Section 6, we recompute and simulate the model with demographics that approximate long-run projections and study the sensitivity of our results to the demographic change. According to the Social Security Administration (SSA), the dependency ratio will exceed 40% by 2080. We use SSAs projected survival rates for 2080 (Bell and Miller 2005) and set the population growth rate at 0.1% that together generate the dependency ratio of 40%.

4.2. Preferences, endowments and technology

4.2.1. Preferences

We assume that the instantaneous utility function takes the form:

$$u(c,\ell) = \log(c) - \chi\frac{\ell^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}, \tag{14}$$

where χ represents the weight on the disutility from work relative to the utility from consumption. When preferences are given by Eq. (14), the IES is constant over the life-cycle and is given by γ. In Section 6, we study the sensitivity of our results to other forms of utility functions. In particular, we simulate the model with separable and non-separable preferences defined over consumption and leisure, both of which are often used in the literature. The subjective discount factor β is calibrated to match a capital–output ratio of 3.0 in the initial steady state. We assume that the capital stock consists of private fixed capital, government capital and the stock of durables. We add the imputed income flows from the last two components of capital stock to measured GNP so that our measurements are consistent with our theory. The parameter χ is chosen to yield the model's fraction of aggregate hours worked equal to its empirical counterpart which is 0.33.

Early estimates of the intertemporal elasticity of substitution in labor using data on men by MaCurdy (1981), Altonji (1986) and Blundell and MaCurdy (1999) are very small, between 0.035 and 0.567. Heckman and MaCurdy (1980) use data on females and estimate a higher intertemporal labor supply elasticity. More recent estimates of IES center around unity; see for example Browning et al. (1999) and Domeij and Floden (2006). Imai and Keane (2004) incorporate human capital accumulation in their overlapping generations model and their estimate using the National Longitudinal Youth Survey of 1979 is 3.8. In our quantitative analysis, we follow the micro labor literature and consider three values of the intertemporal elasticity of substitution in labor $\gamma \in \{0.1, 0.5, 1.0\}$. The value of 0.1 represents the early estimates and the highest value we consider is more representative of the recent estimates.[3,4]

4.2.2. Endowments

Households' labor efficiency depends on two components. The deterministic age-dependent component ε_j is taken from Hansen (1993), which is displayed in Fig. 1. The idiosyncratic component η is specified as a first-order autoregressive process with a persistence parameter $\rho = 0.94$ and the variance of the white noise $\sigma^2 = 0.02$, which lie in the range of estimates in the literature (see, for example, Heathcote et al., 2004). We approximate this continuous process with a five-state, first-order discrete Markov process.

4.2.3. Technology

The income share of capital α is set at 0.40.[5] The depreciation rate δ is $0.082 = \frac{X/Y}{K/Y} - n$, which is implied by the equilibrium law of motion for the capital in the steady state, where we target an investment–output ratio X/Y of 0.28 and a capital–output ratio K/Y of 3.0.[6]

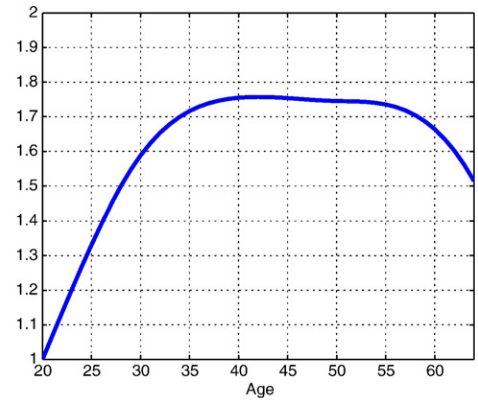

Fig. 1. Age-specific labor efficiency profile from Hansen (1993).

4.3. Social security and fiscal policy

In the initial steady state, the government runs a pay-as-you-go social security program that captures the features of the system in the U.S. We set the social security tax rate τ^{ss} at 10.6% with the maximum taxable amount of $y^{ss} = \$97{,}500$ as it is in the U.S. in 2007. The benefit is a concave piecewise linear function of the average life-time earnings ("AIME"). The marginal replacement rate is 90% for the average earnings up to 20% of the economy's average earnings, above which the replacement rate falls to 32%. For income between 123% and 202% of the economy's average, the replacement rate is 15%. Additional income above 202% of the economy's average does not provide additional pension benefit.[7]

In the initial benchmark economy, we set the government spending G at 20% of output, which is the average ratio of government consumption expenditures and investment to GDP in the post-war period. The ratio of federal debt held by the public to GDP is set at 40%. We assume a consumption tax rate of 5% and a capital income tax rate of 30%. The labor income tax is set so that the government budget constraint is satisfied.

5. Results

5.1. Benchmark simulations

In order to understand how labor supply elasticity affects the quantitative results of social security reform, we numerically characterize three steady-state economies. First, in what we call a benchmark economy, the government operates a pay-as-you-go social security system as described in Section 3. The other two economies differ in the way the fiscal authority deals with the public pension

[3] In an earlier version of the paper, we also considered $\gamma = 2$ and obtained very similar quantitative results.

[4] The calibrated values of the preference parameters are $\beta = \{0.988, 0.986, 0.984\}$ and $\chi = \{217391, 29.7, 9.6\}$ for each model with $\gamma = \{0.1, 0.5, 1.0\}$.

[5] Consistent with the target capital–output ratio, this measure is based on private and public fixed capital and includes the stock of durables, with the service flows from public capital and the stock of durables added to measured output. As part of a sensitivity analysis, we consider a narrower definition of capital and the resulting measure of output. In particular, we exclude the stock of consumer durables and government capital and focus on the private fixed capital. With this new matching of model and NIPA accounts, we calculate the corresponding share of capital in output and the capital–output ratio as 0.32 and 2.43, respectively. Since our main results are not affected, we do not report the results in the paper. They are available from the authors upon request.

[6] We abstract from technological growth since we also consider preference specifications that are not consistent with a balanced growth path. This allows us to experiment with utility functions with alternative values of the coefficient of relative risk aversion that are considered in the literature. Incorporating technological growth does not affect our results in any significant way. A complete set of results with growth is available from the authors upon request. The constant parameter A is normalized so that the average earnings in each model are 1.0 and set at {2.15,2.11,2.07} for each model with $\gamma = \{0.1, 0.5, 1.0\}$.

[7] This is based on the Primary Insurance Amount formula with bend points of \$627 and \$3779, the maximum monthly benefit of \$1939 and national average wage index of \$36,952, all in 2005.

system. The first economy assumes that the benefits and social security tax rate are cut by 50%, which we call half privatization. The second assumes a complete elimination of the unfunded social security system, which we call full privatization. In both economies, the debt-to-GDP ratio is held constant at the benchmark level, and any financial discrepancy between the government's consolidated tax revenues and expenditures are financed by a higher (or lower) labor income tax rate. We follow the literature and keep all experiments revenue-neutral by assuming a fixed level of exogenous government purchases *G* across three economies.

We compare the effects of social security reforms along two dimensions. First, we describe the effects on aggregate macroeconomic indicators. Second, we document the effects on the allocation of consumption, assets, and labor supply over the life-cycle. In all cases, we consider various labor supply elasticities to investigate the role played by this important preference parameter on both macro and micro results.

Table 1 summarizes the aggregate effects of the social security reforms when we employ preferences of the form in Eq. (14) with the Frisch elasticity of labor supply γ. We compute three steady-state economies for each of three alternative values of Frisch labor supply elasticity, γ {0.1,0.5,1.0}. For each model, we recalibrate the parameters of the model in order to match the same aggregate statistics that we described in Section 4.

Table 1
Aggregate effects of social security reforms.

IES γ	0.1	0.5	1.0
PAYGO system (benchmark)			
Interest rate (%)	5.1%	5.1%	5.1%
Labor income tax: $\tau^{\ell}+\tau^{ss}$ (%)	32.8%	32.9%	32.9%
Half privatization			
Capital	+10.8%	+10.3%	+9.8%
Labor	+0.06%	+0.16%	+0.25%
Average work hours	+0.03%	+0.05%	+0.11%
Wage	+4.2%	+3.9%	+3.7%
Interest rate (%)	4.3%	4.3%	4.4%
Labor income tax: $\tau^{\ell}+\tau^{ss}$ (%)	27.0%	27.0%	27.0%
Long-run welfare: CEV (%)	+5.0%	+5.2%	+5.4%
Full privatization			
Capital	+26.2%	+24.9%	+23.6%
Labor	+0.09%	+0.22%	+0.35%
Average work hours	+0.01%	−0.04%	+0.04%
Wage	+9.7%	+9.2%	+8.7%
Interest rate (%)	3.4%	3.4%	3.5%
Labor income tax: $\tau^{\ell}+\tau^{ss}$ (%)	21.2%	21.3%	21.3%
Long-run welfare: CEV (%)	+9.1%	+9.7%	+10.2%

5.1.1. Aggregate effects of a typical social security reform

Before we analyze the role played by the intertemporal elasticity of substitution in labor supply in affecting the long-run effects of social security reforms, it will be useful to describe the typical effects of social security reform for a given value of the IES. We focus on the column labeled *0.5* (for IES) of Table 1 that describes the effects of two possible reforms in which the social security policy is different.

When we use an IES equal to *0.5*, a reform of the unfunded retirement system delivers long-run gains. Half privatization leads to a decrease in the combined labor income tax rate $\tau^{\ell}+\tau^{ss}$, from the initial *32.9%* to *27.0%*. Since the households are now forced to partially support their own old age consumption, they increase their private saving and the capital stock shows an increase of *10.3%*.[8] However, the impact on labor market aggregates is very small. There is a marginal increase in average hours worked at *0.05%* and the rise in the aggregate labor supply is only *0.16%*. Aggregate capital becomes more abundant relative to the labor input of the economy and the interest rate falls from *5.1%* to *4.3%*, while the wage rate increases by *3.9%*.

Full privatization amplifies the effects on private saving and aggregate capital stock since the households are now entirely responsible for their old age consumption. The capital stock increases by *24.9%*. The combined tax rate on labor supply decreases further down to *21.3%*, mostly due to the elimination of the *10.6%* social security tax. There is a negligibly small change in average hours worked and aggregate labor supply. These numerical findings are similar to those in the previous literature that study the effects of social security reform towards a fully-funded system.

5.2. Labor supply elasticity and the effects of reform

We now turn our attention to the effects of reforms across models with different labor supply elasticities and examine Table 1 in its entirety. We find that the magnitude of the responses in aggregate labor supply and average hours of work are surprisingly small and similar across experiments. With full privatization, for example, the change in average work hours lies in the tight range of −0.04% to +0.04% and the aggregate labor supply increases by very little, in the range of 0.09% to 0.35%. These numerical findings are in line with the U.S. facts documented by McGrattan and Rogerson (2004). Using data from the U.S. Bureau of the Census decennial censuses from 1950 to 2000, McGrattan and Rogerson show that "… there has been a negligible change in average hours per person at the aggregate level." Our model's result is analogous to this observation, regardless of the value of the IES, and we explore this point further below.

Hidden, however, behind the relatively small effects of different elasticities on aggregate labor supply are fairly large effects on the distribution of hours over the life-cycle. Left panels of Fig. 2 show the age-hours profiles for values of IES equal to 0.1, 0.5 and 1.0, respectively. For example, with a labor supply elasticity of 1.0 which is close to the ones implied by calibrated non-separable utility functions common in the literature on social security reform, full privatization yields a very large reallocation of hours over the life-cycle, significantly reducing them at younger ages and raising them at mid- to old-ages. This finding suggests that the introduction of an unfunded social security system in the U.S. may explain, in part, the observed reallocation of hours over the life-cycle. Chart 1 in McGrattan and Rogerson (2004) (pp. 26) indicates that there has been a significant shift of hours worked from late ages before retirement to earlier ages over the life-cycle. In the bottom left plot of Fig. 2 the patterns of work hours in the profile labeled 'PAYGO system' and 'Full Privatization' seem to be consistent with the hours profiles for the cohorts born in 1976–85 and 1866–75, respectively, that are presented in Chart 1 in McGrattan and Rogerson (2004).[9]

It will be useful to explore the reasons behind this reallocation of hours over the life-cycle and its sensitivity to the IES. Notice that there is a very small quantitative difference in the change in the interest rate across different models, which ranges from 3.4% under full privatization with an IES of 0.1 to 3.5% with an IES equal to 1.0, relative to 5.1% in the benchmark economy. The decrease in the interest rate induces the households to choose a much flatter path of consumption as optimal, a path that is similar across different labor supply elasticities as shown in the right panels of Fig. 2. The same economic forces also flatten the age-hours profile, but the impact on the allocation of hours can vary across models with different elasticities. One can describe intuition

[8] These results are broadly consistent with recent microeconometric evidence on the increase in private saving after social security reform in Italy and United Kingdom, as presented by Attanasio and Brugiavini (2003) and Attanasio and Rohwedder (2003).

[9] McGrattan and Rogerson (2004) also document a decline of hours worked among the younger cohorts of age 20–25. This may be due to the increase over time in educational attainment and on-the-job skill accumulation. Our model abstracts from these factors and therefore we do not get this decline.

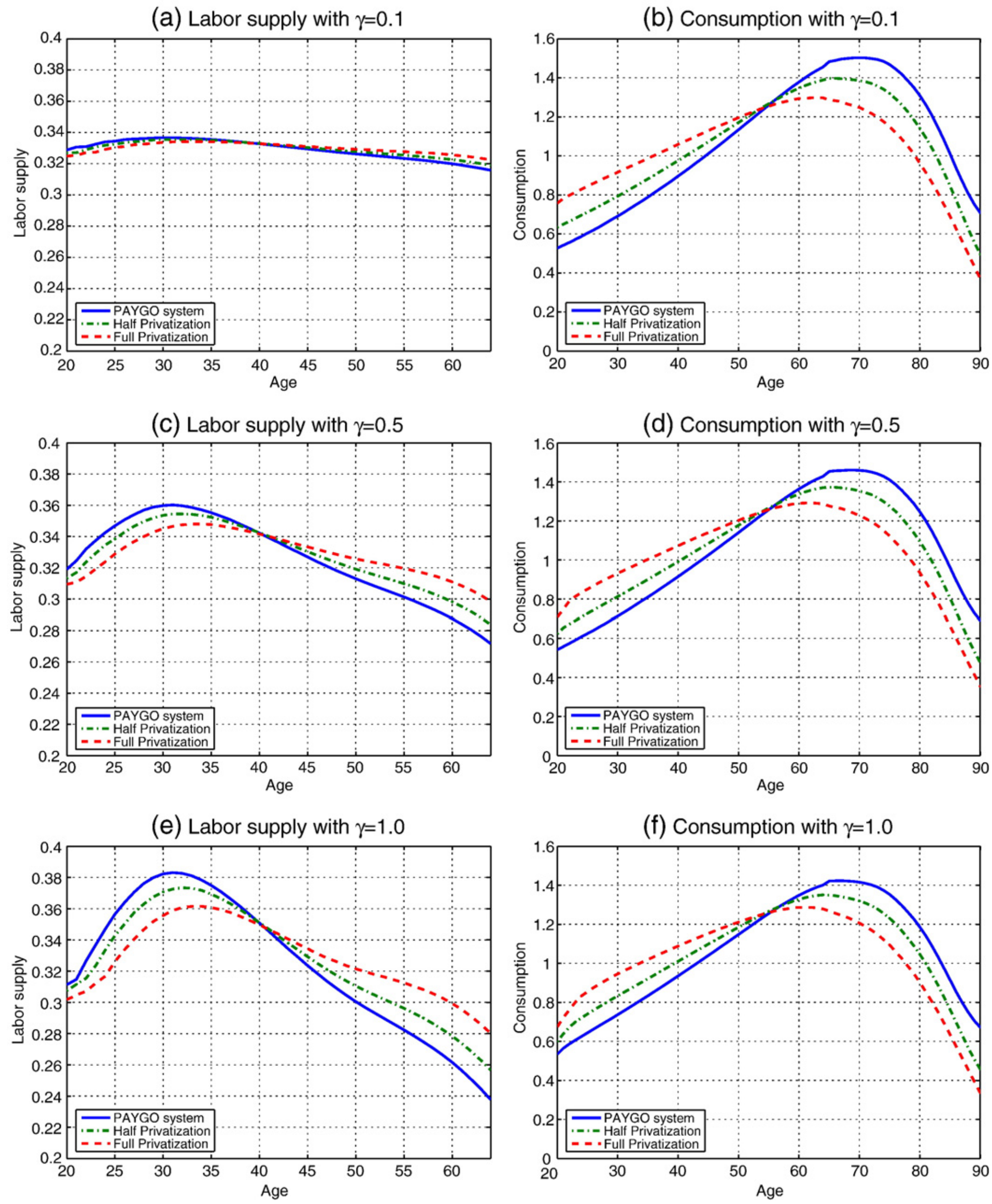

Fig. 2. Labor supply and consumption over the life-cycle: the level of consumption is normalized by the average consumption in each model with the PAYGO system.

behind this result by examining a simplified version of the first-order conditions, ignoring productivity uncertainty, the marginal effect on the social security benefits, and borrowing constraints. The intertemporal and intratemporal optimality conditions are given as:

$$u'\left(c_j\right) = u'\left(c_{j+1}\right)\beta s_j\{1 + r(1-\tau^a)\}$$
$$u'\left(c_j\right)\left(1-\tau^{\ell}\right)\varepsilon_j w = v'\left(\ell_j\right),$$

where $u'(c_j)$ denotes the marginal utility from consumption and $v'(\ell_j)$ the marginal disutility of work, at age j. Using the preference specification we employed and combining the two equations, we have:

$$\frac{\varepsilon_{j+1}}{\varepsilon_j}\left(\frac{\ell_j}{\ell_{j+1}}\right)^{\frac{1}{\gamma}} = \beta s_j\{1 + r(1-\tau^a)\} = \frac{c_{j+1}}{c_j}.$$

Given our calibration of β to match the common capital–output ratio in each model, $\beta\{1+r(1-\tau^a)\} > 1.0$ for all the models that we considered.[10]

Therefore a lower interest rate (resulting from the increase in the capital–labor ratio due to social security reform) will flatten both the age-consumption and the age-hours profiles, but the latter with a greater intensity if the IES γ is larger. In order to highlight the difference, Fig. 3 plots the ratio of hours profiles in reform economies to that of the benchmark economy. The labor supply of households in a

[10] The value of β that is calibrated to match the common capital–output ratio in the benchmark of different models lies in the narrow range between 0.988 with $\gamma = 0.1$ and 0.984 with $\gamma = 1.0$. The conditional survival probabilities are high and close to unity during working ages, lying above 0.99 until age 62. Therefore the optimal growth rate of consumption before retirement is positive even after taking into account the additional discounting by death probabilities.

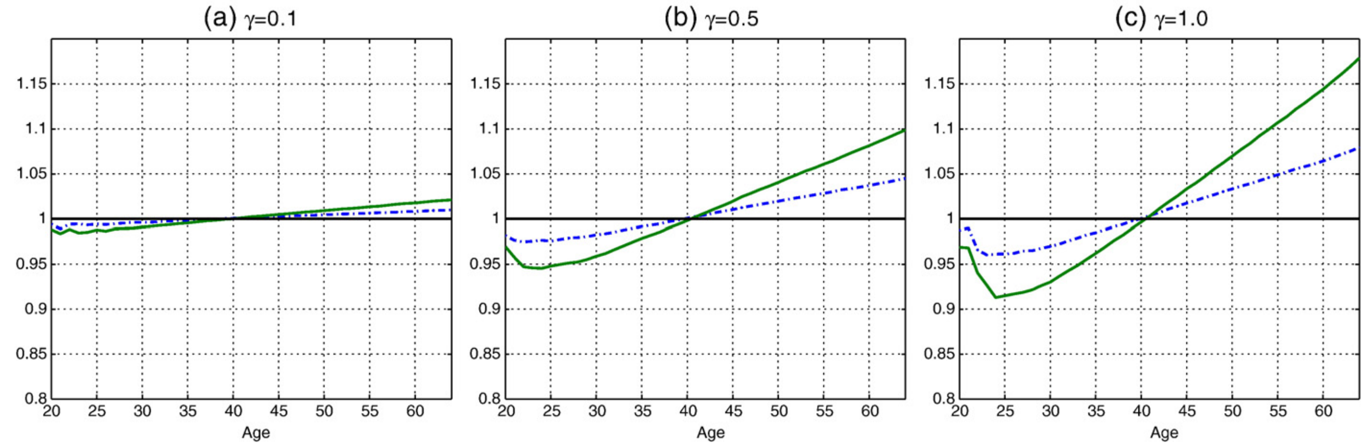

Fig. 3. Changes in labor supply over the life-cycle: solid lines represent the labor supply under full privatization relative to the benchmark PAYGO system and dashed lines represent half privatization.

year before retirement is higher by as much as 18% in the case of $\gamma = 1.0$ under full privatization. This is a very large reallocation of hours over the life-cycle in response to social security reform. The change, however, is 10% with $\gamma = 0.5$ and only a few percentage points with $\gamma = 0.1$. The rise in aggregate labor supply is larger than the change in average work hours, since those mid- to old-age agents who increase work hours are more productive and contribute more per hour than the younger agents who reduce work hours, raising the average productivity of workers.

Although the effect on the age-consumption profile is nearly identical across models indexed by γ, the increase in aggregate capital is larger with lower elasticities. To understand the reason, note that the need to save and finance one's retirement consumption in the face of reduced or eliminated public pension is met by the combination of lower consumption before reaching the retirement age and earnings by additional market hours. With a higher labor supply elasticity, the second adjustment is used more extensively as households are more willing to intertemporally substitute labor. As we saw in Fig. 2, with a higher labor supply elasticity, households work and earn more at older ages to meet the additional need for retirement savings. Therefore compared to the case of a very low elasticity where the hours profile barely changes, there is less need for them to start accumulating wealth at younger ages by cutting back the consumption. Put differently, the life-cycle saving motive becomes more pronounced when the labor supply elasticity is small and this causes the aggregate capital stock to increase by a larger percentage in response to social security reform for small values of IES.

In Table 1, the last rows of the privatization experiments present the long-run welfare effects of the two social security reforms relative to the benchmark economy with the PAYGO social security system. They are computed as consumption equivalent variations from the ex-ante perspective of a newborn household in the economy. The welfare analysis is based on the comparison of the two steady states and the costs associated with the transition are not considered. It measures the percentage increase in consumption across all possible states of the benchmark economy that makes the household indifferent between the economies with and without reform. A positive number implies households are better off under the reform economy. With a higher labor supply elasticity, the age-hours profile responds more to the change in the interest rate without incurring as much utility cost as it would under the low labor supply elasticity. Ex-post, the distribution of work hours across ages is smoother and flatter compared to the benchmark economy, which because of the convexity in the disutility of labor, improves welfare.

6. Sensitivity analysis and extensions

In this section we conduct a sensitivity analysis to document how our quantitative findings are affected if different utility functions are used and when we allow for a change in demographics.[11]

6.1. Different preference specifications

The period utility function (14) used in the previous section is quite common in the applied labor literature but not in the studies of social security reforms using dynamic macro models. However, it has the convenient property that the IES is constant over the life-cycle. In this section, we consider three alternative forms of period utility functions and study the sensitivity of our results to these preference specifications.

The first is a separable preference defined over consumption in log and leisure, commonly used in the real business cycle literature, given as:

$$u(c, 1-\ell) = \log(c) + \psi \frac{(1-\ell)^{1-\theta}}{1-\theta}, \tag{15}$$

which we call 'separable preference II'. We call 'separable preference I' the baseline separable preference defined over consumption in log and labor supply as in Eq. (14). In Eq. (15), the relative utility weight on leisure is given by the parameter ψ, which is calibrated so that households allocate one third of their disposable time to market work on average as before. The IES is given by $\frac{1}{\theta}\frac{1-\ell}{\ell}$ and it varies over the life-cycle as a function of leisure relative to work hours. We experiment with two values of θ at 4.0 and 2.0, which imply average IES values of 0.5 and 1.0, respectively.

The second utility function is defined over labor supply as in the baseline separable preference I, but we explore alternative values of the

[11] We also investigated the role of borrowing constraints in influencing the degree to which different values of the IES affect labor supply. When we relax the borrowing constraint and allow individuals to borrow up to one year's worth of average income at any age until retirement, reform effects are not only qualitatively but also quantitatively very similar to the model with a tight borrowing constraint at zero. The main reason is that agents in our model are engaged in private saving from the early years of their life, not only for life cycle reasons, but also for precautionary reasons to insure themselves against idiosyncratic income and longevity risks. Therefore, there are very few agents who are borrowing constrained in our original model. These results are also available from the authors upon request.

Table 2
Aggregate effects of social security reforms: alternative preferences.

Preference	Separable II		Separable III			Non-separable	
θ	4.0	2.0	–	–	–	–	–
σ	–	–	0.5	1.0	1.5	1.0	2.0
IES	0.5	1.0	0.5	0.5	0.5	2.0	1.34
CRRA	1.0	1.0	0.5	1.0	1.5	1.0	1.34
PAYGO system (benchmark)							
Interest rate (%)	5.1%	5.1%	5.1%	5.1%	5.1%	5.1%	5.1%
Labor income tax: $\tau^{\ell}+\tau^{ss}$ (%)	32.8%	32.9%	32.9%	32.9%	32.8%	32.9%	32.8%
Half privatization							
Capital	+10.6%	+10.4%	+6.1%	+10.3%	+14.1%	+10.0%	+15.1%
Labor	+0.4%	+0.7%	+0.8%	+0.16%	−0.4%	+1.0%	+1.8%
Average work hours	+0.3%	+0.6%	+0.9%	+0.05%	−0.7%	+0.8%	+1.5%
Wage	+3.9%	+3.7%	+2.1%	+3.9%	+5.6%	+3.5%	+5.0%
Interest rate (%)	4.4%	4.4%	4.7%	4.3%	4.1%	4.4%	4.2%
Labor income tax: $\tau^{\ell}+\tau^{ss}$ (%)	26.9%	26.9%	27.2%	27.0%	26.9%	26.8%	26.3%
Long-run welfare: CEV (%)	+5.3%	+5.6%	+4.3%	+5.2%	+6.0%	+6.0%	+7.1%

coefficient of relative risk aversion (CRRA), using a general isoelastic function of consumption given as:

$$u(c,\ell)=\frac{c^{1-\sigma}}{1-\sigma}-\chi\frac{\ell^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}. \tag{16}$$

We call this specification as ‘separable preference III.’ We experiment with two additional values of the relative risk aversion σ at 0.5 and 1.5 and set the Frisch elasticity of labor supply γ at 0.5. Note that with $\sigma=1.0$, the specification is identical to the one with the baseline model with the results for IES = 0.5 in Table 1.

The third utility function we consider in this section defines preferences that are non-separable in consumption and leisure,

$$u(c,1-\ell)=\frac{\left[c^{\nu}(1-\ell)^{1-\nu}\right]^{1-\sigma}}{1-\sigma}. \tag{17}$$

The parameter ν represents the utility weight on consumption, calibrated to match the common target of work hours at 0.33. We compute the model with two values of σ at 1.0 (log) and 2.0, which imply the coefficient of relative risk aversion given as $1-\nu(1-\sigma)$ at 1.0, the same as that in the baseline model, and 1.34, respectively. This utility function has been used in most general equilibrium studies of social security reform. The Frisch elasticity in this case also varies over the life-cycle and depends on the ratio of leisure to work hours over the life-cycle.[12]

Once we calibrate models with different utility functions and parameter values to the common calibration targets, we repeat the simulation of social security reform. Our results of half privatization are summarized in Table 2.

The first two columns of Table 2 show the results under separable preference II defined over consumption and leisure. If we compare these with the results in Table 1, the aggregate effects are surprisingly similar across different utility functions. The increases in the aggregate capital stock are 10.6% and 10.4% under IES values of 0.5 and 1.0 here, respectively, and the corresponding numbers under separable preference I from Table 1 are 10.3% and 9.8%. The wage and the labor income tax change by very similar magnitudes and the welfare effects are comparable under the two different forms of separable preferences. As before, the effect on aggregate labor supply is very small, with increases of only 0.4% and 0.7%, respectively.

There is a large difference, however, in how the hours profile responds to social security reform. Note that with the form of preferences in Eq. (15), IES is not constant over the life-cycle and declines in market hours. Labor supply typically falls in pre-retirement ages in life-cycle models, which is also the case in our calibrated general equilibrium model. Therefore, the life-cycle effects that we observe under the baseline preferences with a constant IES in Eq. (14) will be magnified under these preferences. The reallocation of hours worked from early working ages to later working ages will be much larger under separable preferences II since individuals supply labor more elastically when they are closer to retirement than when they are prime aged. The left panel of Fig. 4 shows the labor supply elasticity over the life-cycle under separable preference II with $\theta=2.0$. Hours worked just before retirement are much higher under the reform, but the effects under separable preferences II are even larger as shown in the left panel of Fig. 5.[13]

The middle three columns of Table 2 summarize the results with separable preference III with σ at 0.5, 1.0 and 1.5, where the middle one ($\sigma=1.0$, in log) is the baseline specification that we studied in Section 5.1. With a higher value of the CRRA and a lower intertemporal elasticity of substitution, households are less willing to accept a decline of consumption at older ages due to the social security reform and respond to it by raising their saving more aggressively. The capital stock increases by 14.1% in the model with $\sigma=1.5$, while it goes up by only 6.1% with $\sigma=0.5$. As a result, the factor prices respond by more with a higher risk aversion, since the aggregate labor does not vary much across experiments. As we saw in Section 2, with the CRRA less than unity, a rise in the wage rate tends to increase the labor supply and the opposite is the case with the CRRA exceeding one. As shown in Table 2, the average work hours vary by no more than a percentage point across different values of the CRRA, but the change declines in the degree of the risk aversion.

The last two columns of Table 2 show the results with non-separable preferences with the value of σ at 1.0 (log) and 2.0. The former value represents the coefficient of relative risk aversion at 1.0, corresponding to that under the separable preferences I and II that we considered above. If we compare the results of $\sigma=1.0$ with those of separable

[12] The Frisch elasticity is given as $\frac{1-\ell}{\ell}\frac{1-\nu(1-\sigma)}{\sigma}$ and takes a value of 2.0 and 1.34 on average when σ is set at 1.0 and 2.0, respectively.

[13] Note that the scale for the vertical axis is different from Fig. 3. A complete set of numerical results (half and full privatization) under alternative utility functions and parameters and accompanying figures is available upon request from the authors, which are not displayed here to save space.

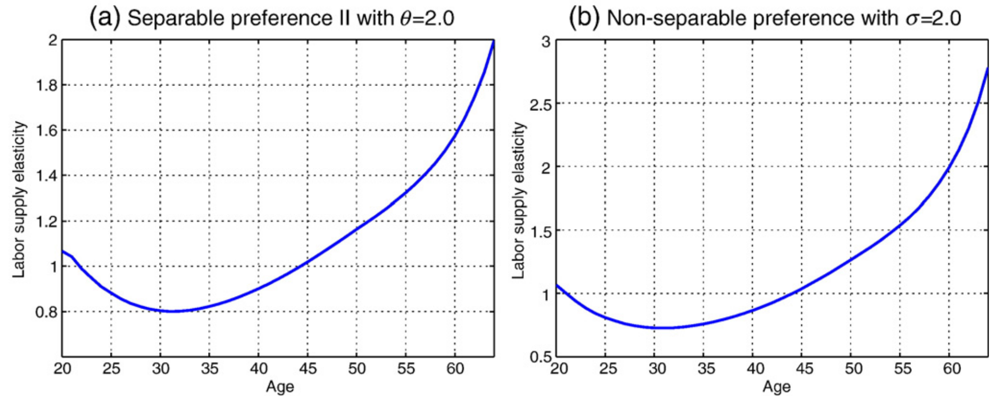

Fig. 4. Labor supply elasticity over the life-cycle.

preferences, the aggregate effects are quantitatively similar. In the half privatization reform, aggregate capital stock increases by 10.0% and the combined labor income tax rate falls to 26.8%. The welfare effects are also similar, at 6.0% in consumption equivalent variation. The change in aggregate labor is small as in other specifications, at 1.0%.

As shown in the last column of Table 2, when σ is set at 2.0 and the relative risk aversion is higher (and intertemporal elasticity of substitution is lower), the aggregate effects are much larger. Aggregate capital increases by 15.1%, relative to 10.0% with $\sigma=1.0$. As we saw in the separable preference III, with a lower intertemporal elasticity of substitution, the effect on the capital is larger since households' consumption profile responds less to a decline in the interest rate. The labor supply will also respond by more than that in the case with lower σ. The non-separable preference of the form (17) implies a complementarity between consumption and labor (i.e. $u_{\ell,c}>0$) when σ takes a value greater than unity. The rise in consumption under reforms will reduce the disutility from work and induce more work effort, contributing to a larger increase in the aggregate labor supply. Due to the additional utility from a large increase in consumption, which offsets the higher disutility from more work, the welfare gain is larger with $\sigma=2$.

6.2. Effects of demographic change and social security reforms

In this section, we study the sensitivity of our results under different demographic structures. In particular, we investigate how our findings about the effects of different values for IES on the quantitative predictions of social security reform will change when we incorporate the projected aging of the population into the models. Instead of assuming the stationary demographic structure in 2005 summarized by current survival rates and population growth, we run the reform experiments using the demographic variables for 2080, based on projected conditional survival rates and the predicted 2080 dependency ratio. As mentioned in Section 4, we use Bell and Miller (2005) for the projected survival rates and set the population growth rate at 0.1%, which together with the increased longevity implies an old-age dependency ratio of 40% in 2080, nearly twice as large as the ratio used in the baseline simulations. In what follows, we call this new economy with more elderly and higher longevity as the economy with aging.

Before analyzing the effects of social security reforms, we will briefly describe the effects of the demographic change by itself. Table 3 summarizes the changes in the aggregate statistics in the economy with aging, where the current pay-as-you-go social security system is maintained.

The higher dependency ratio increases the cost of providing pension benefits at the benchmark levels and the combined labor income tax will rise by 8 percentage points to 41%. Since the fraction of working population falls, per capita labor supply in the economy falls significantly, by about 11 to 12 percentage points. The decrease in the interest rate, caused by a large increase in the aggregate capital–labor ratio, will lower the optimal growth rate of consumption and flatten its life-cycle profile. As a result, private saving will fall and reduce the aggregate capital stock compared to the benchmark economy. Also note that despite the large increase in the labor income tax rate, average work hours increase although the magnitude is small, at most 1.5% with $\gamma=1.0$. The substitution effect due to the lower after-tax wage rate is offset by the income effect due to the lower level of consumption. Similar to the effect of privatization that we examined in previous sections, a lower interest rate will flatten not only the consumption profile but also the age-hours profile, and more productive households in older ages will provide a higher work effort.

Now we turn our attention to the effect of social security reform in the economy with aging. Table 4 summarizes the effects of half

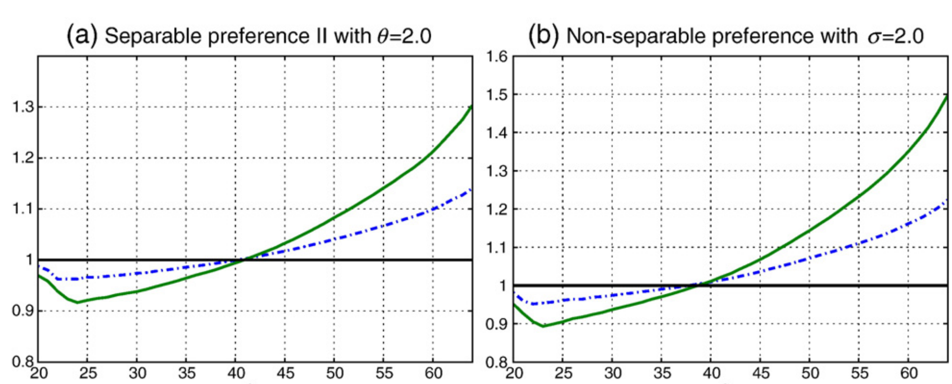

Fig. 5. Changes in labor supply over the life-cycle: solid lines represent the labor supply under full privatization relative to the benchmark PAYGO system and dashed lines half privatization.

privatization, where the numbers are expressed in terms of the distance from the economy with aging but with unfunded social security, the economy that we just studied above.

If we compare the results to those in the benchmark economy without aging in Table 1, qualitative effects on aggregate variables and the impact of different elasticities are very similar. The magnitude, however, is very different. Half privatization raises aggregate capital by about 17%, compared to 10 to 11% in the benchmark economy without aging. Households live longer in this economy and if the public pension is cut in half, they would have to accumulate much more savings for retirement on their own. Due to the massive increase in aggregate capital, the interest rate will decline by more, which flattens the profiles of consumption and labor supply further compared to the benchmark. Due to the large positive effect on output and consumption as a result of the larger capital stock of the economy, the long-run welfare gain is much larger as well.

7. Conclusion

The intertemporal elasticity of substitution in labor is a crucial parameter in determining the response of labor supply to changes in policy and factor prices. Early micro labor estimates by Altonji (1986) and MaCurdy (1981) suggest that labor supply is quite inelastic. More recent structural estimates utilized departures from the earlier representative agent models and also incorporated data other than prime-age males, producing estimates that are near unity. In the business cycle literature, early work by Kydland and Prescott (1982) relied on a high IES as the propagation mechanism of the business cycles in their neoclassical theory. Recent work by Prescott (2004) also argues for a high IES to explain the difference in average market hours between U.S. and European countries.

In this paper, we explore the role of IES in shaping the quantitative results of social security reform. We start with a two-period, partial equilibrium, deterministic overlapping generations model. Our analytical results using a separable utility function extend previous research and show that the labor supply response to a price change depends on the sizes of substitution and income effects. For the special case of logarithmic utility on consumption there is no effect on labor supply when wage rises; a decrease in interest rate induces less work when young and more work when old. For more general utility functions, including non-separable preferences, and in a general equilibrium framework, analytical investigation is not feasible. Therefore, we consider a quantitative general equilibrium model populated with overlapping generations of individuals who face uninsurable income risk, mortality risk and borrowing constraints. Individuals choose hours worked until the mandatory retirement age, in addition to the usual consumption-saving decision over the life-cycle. We consider three classes of preferences that have been used in the applied labor, real business cycle, and public finance strands of the literature. For each period utility function, we evaluate the quantitative results of social security reform using a range of values for the IES commonly estimated and used in previous research. In each case, we calibrate the model to the same aggregate targets consistent with features of the U. S. economy over the past five decades.

We have two main quantitative findings. First, a particular period utility function and the value for the IES have a negligible impact on

Table 3
Effects of the demographic change relative to the benchmark economy without aging.

IES γ	0.1	0.5	1.0
Capital (per capita)	−5.0%	−4.7%	−4.7%
Labor (per capita)	−11.9%	−11.1%	−10.8%
Average work hours	+0.4%	+1.1%	+1.5%
Wage	+3.0%	+2.8%	+2.6%
Interest rate (%)	4.5%	4.6%	4.6%
Labor income tax: $\tau^{\ell}+\tau^{ss}$ (%)	41.1%	41.1%	41.1%

Table 4
Effects of a reform in the economy with aging.

IES γ	0.1	0.5	1.0
Half privatization			
Capital	+17.4%	+17.4%	+17.1%
Labor	+0.2%	+0.7%	+1.1%
Average work hours	+0.2%	+0.6%	+1.0%
Wage	+6.6%	+6.3%	+6.0%
Interest rate (%)	3.4%	3.4%	3.5%
Labor income tax: $\tau^{\ell}+\tau^{ss}$ (%)	31.1%	30.9%	30.8%
Long-run welfare: CEV (%)	+10.7%	+11.3%	+12.0%

the aggregate effects of social security reform. In all cases considered, reform results in an increase in the capital stock which is within a percentage point or two over the range of IES coefficients used. Aggregate labor supply is essentially unchanged in the long-run, consistent with the earlier findings of the overall canceling out of income and substitution effects in the long-run. However, reform generates a significant reallocation of hours worked over the life-cycle. This brings us to the second main finding of the paper. The increase in the capital–labor ratio fueled by social security reform lowers the real interest rate and flattens the life-cycle profile of labor supply. Individuals shift work hours from younger years to older years before they retire. This reallocation is quantitatively more significant with a higher IES. Therefore the value of IES used in analyzing social security reform has important implications on how individuals allocate hours over their life-cycle and the composition of labor supply of the economy.

Finally, although our model has focused on the adjustment of labor supply at the intensive margin, we conjecture that the value of IES may also influence the participation and retirement decisions. Further investigating the effects on both intensive and extensive margins calls for a model that endogenizes participation decisions. We will investigate these important issues in an ongoing research.

Appendix A. The effect of a change in the interest rate

Rewriting the optimal choice of the labor supply ℓ_1 in Eq. (8),

$$\begin{aligned}\ell_1 &= w^{\frac{\gamma(1-\sigma)}{1+\sigma\gamma}}\left\{\left[\varepsilon_1^{1+\gamma}+\frac{\varepsilon_2^{1+\gamma}}{\beta^{\gamma}R^{1+\gamma}}\right]\left(1+R^{\frac{1-\sigma}{\sigma}}\beta^{\frac{1}{\sigma}}\right)^{-1}\right\}^{-\frac{\sigma\gamma}{1+\sigma\gamma}}\varepsilon_1^{\gamma}\\ &= w^{\frac{\gamma(1-\sigma)}{1+\sigma\gamma}}\varepsilon_1^{\gamma}M^{-\frac{\sigma\gamma}{1+\sigma\gamma}},\end{aligned} \tag{18}$$

where:

$$M\equiv\left[\frac{\varepsilon_2^{1+\gamma}}{\beta^{\gamma}R^{1+\gamma}}\right]\left(1+R^{\frac{1-\sigma}{\sigma}}\beta^{\frac{1}{\sigma}}\right)^{-1}. \tag{19}$$

$$\begin{aligned}\frac{\partial\ell_1}{\partial R} &= w^{\frac{\gamma(1-\sigma)}{1+\sigma\gamma}}\varepsilon_1^{\gamma}\frac{\partial M^{-\frac{\sigma\gamma}{1+\sigma\gamma}}}{\partial R}\\ &= w^{\frac{\gamma(1-\sigma)}{1+\sigma\gamma}}\varepsilon_1^{\gamma}\left(-\frac{\sigma\gamma}{1+\sigma\gamma}\right)M^{-\frac{\sigma\gamma}{1+\sigma\gamma}-1}\left[-(1+\gamma)\frac{\varepsilon_2^{1+\gamma}}{\beta^{\gamma}R^{2+\gamma}}\left(1+R^{\frac{1-\sigma}{\sigma}}\beta^{\frac{1}{\sigma}}\right)^{-1}\right.\\ &\quad\left.-\frac{1-\sigma}{\sigma}\left(\varepsilon_1^{1+\gamma}+\frac{\varepsilon_2^{1+\gamma}}{\beta^{\gamma}R^{1+\gamma}}\right)\left(1+R^{\frac{1-\sigma}{\sigma}}\beta^{\frac{1}{\sigma}}\right)^{-2}R^{\frac{1-2\sigma}{\sigma}}\beta^{\frac{1}{\sigma}}\right]\\ &= \ell_1\left(-\frac{\sigma\gamma}{1+\sigma\gamma}\right)\left[-(1+\gamma)\left(\varepsilon_1^{1+\gamma}+\frac{\varepsilon_2^{1+\gamma}}{\beta^{\gamma}R^{1+\gamma}}\right)^{-1}\frac{\varepsilon_2^{1+\gamma}}{\beta^{\gamma}R^{2+\gamma}}-\frac{1-\sigma}{\sigma}\frac{\beta^{\frac{1}{\sigma}}R^{\frac{1-2\sigma}{\sigma}}}{1+\beta^{\frac{1}{\sigma}}R^{\frac{1-\sigma}{\sigma}}}.\right]\end{aligned}$$

The sign of the derivative is positive if $\sigma\leq 1$ but it is ambiguous if $\sigma>1$.

Similarly, rewriting the optimal choice of the labor supply ℓ_2 in Eq. (8),

$$\ell_2 = w^{\frac{\gamma(1-\sigma)}{1+\sigma\gamma}}\left\{\left[\varepsilon_1^{1+\gamma} + \frac{\varepsilon_2^{1+\gamma}}{\beta^\gamma R^{1+\gamma}}\right]\left(1 + R^{\frac{1-\sigma}{\sigma}}\beta^{\frac{1}{\sigma}}\right)^{-1}\right\}^{-\frac{\sigma\gamma}{1+\sigma\gamma}}\left(\frac{\varepsilon_2}{\beta R}\right)^\gamma$$
$$= w^{\frac{\gamma(1-\sigma)}{1+\sigma\gamma}} M^{-\frac{\sigma\gamma}{1+\sigma\gamma}}\left(\frac{\varepsilon_2}{\beta R}\right)^\gamma$$

where M is defined as in Eq. (19).

Taking a partial derivative of ℓ_2 with respect to the interest rate R,

$$\frac{\partial \ell_2}{\partial R} = w^{\frac{\gamma(1-\sigma)}{1+\sigma\gamma}}\left(\frac{\varepsilon_2}{\beta}\right)^\gamma\left[-\frac{\sigma\gamma}{1+\sigma\gamma}M^{-\frac{\sigma\gamma}{1+\sigma\gamma}-1}\frac{\partial M}{\partial R}R^{-\gamma} - \gamma M^{-\frac{\sigma\gamma}{1+\sigma\gamma}}R^{-\gamma-1}\right] \quad (20)$$

The first term in the square bracket of Eq. (20) is positive if $\sigma \le 1$, while the second term is unambiguously negative. Therefore, the sign of the derivative is undetermined.

References

Altonji, J.G., 1986. Intertemporal substitution of labor supply: evidence from microdata. Journal of Political Economy 94 (3), 176–215.
Attanasio, O.P., Brugiavini, A., 2003. Social security and households' saving. Quarterly Journal of Economics 118 (3), 1075–1119.
Attanasio, O.P., Rohwedder, S., 2003. Pension wealth and household saving: evidence from pension reforms in the UK. American Economic Review 93 (5), 1499–1521.
Bell, F.C. and Miller, M.L., (2005). Life tables for the United States social security area 1900–2100. Office of the Chief Actuary, Social Security Administration, Actuarial Study 116.
Blundell, R., MaCurdy, T., 1999. Labor supply: a review of alternative approaches. In: Ashenfelter, O., Card, D. (Eds.), Handbook of Labor Economics, vol. 3A. North-Holland, Amsterdam. Chapter 27.
Boldrin, M., Rustichini, A., 2000. Political equilibria with social security. Review of Economic Dynamics 3 (1), 41–78.
Browning, M., Hansen, L.P., Heckman, J.J., 1999. Micro data and general equilibrium models. In: Taylor, J.B., Woodford, M. (Eds.), Handbook of Macroeconomics, vol. 1A. North-Holland, Amsterdam. Chapter 8.
Casamatta, G., Cremer, H., Pestieau, P., 2002. The political economy of social security. Scandinavian Journal of Economics 102 (3), 503–522.
Chetty, R., 2006. A new method of estimating risk aversion. American Economic Review 96 (5), 1821–1834.
Conesa, J.C., Krueger, D., 1999. Social security with heterogeneous agents. Review of Economic Dynamics 2 (4), 757–795.
Cooley, T., Soares, J., 1999. A positive theory of social security based on reputation. Journal of Political Economy 107 (1), 135–160.
De Nardi, M., İmrohoroğlu, S., Sargent, T.J., 1999. Projected U.S. demographics and social security. Review of Economic Dynamics 2 (3), 575–615.
Domeij, D., Floden, M., 2006. The labor–supply elasticity and borrowing constraints: why estimates are biased. Review of Economic Dynamics 9 (2), 242–262.
Feldstein, M., 1985. The optimal level of social security benefits. Quarterly Journal of Economics 100 (2), 303–320.
Fuster, L., İmrohoroğlu, A., İmrohoroğlu, S., 2007. Elimination of social security in a dynastic framework. Review of Economic Studies 74 (1), 113–145.
Hansen, G.D., 1993. The cyclical and secular behaviour of the labour input: comparing efficiency units and hours worked. Journal of Applied Econometrics 8 (1), 71–80.
Heathcote, J., Storesletten, K., Violante, G.L., 2004. The macroeconomic implications of rising wage inequality in the United States. Working Paper, New York University.
Heckman, J., MaCurdy, T., 1980. A life-cycle model of female labor supply. Review of Economic Studies 47 (1), 47–74.
Huang, H., İmrohoroğlu, S., Sargent, T.J., 1997. Two computations to fund social security. Macroeconomic Dynamics 1, 7–44.
Hubbard, R.G., Judd, K., 1987. Social security and individual welfare: precautionary saving, borrowing constraints, and the payroll tax. American Economic Review 177 (4), 630–646.
Imai, S., Keane, M.P., 2004. Intertemporal labor supply and human capital accumulation. International Economic Review 45 (2), 601–641.
İmrohoroğlu, A., İmrohoroğlu, S., Joines, D., 1995. A life cycle analysis of social security. Economic Theory 6 (1), 83–114.
İmrohoroğlu, A., İmrohoroğlu, S., Joines, D., 2003. Time inconsistent preferences and social security. Quarterly Journal of Economics 118 (2), 745–784.
Kotlikoff, L., Smetters, K., Walliser, J., 1999. Privatizing social security in the united states—comparing the options. Review of Economic Dynamics 2 (3), 532–574.
Kydland, F.E., Prescott, E.C., 1982. Time to build and aggregate fluctuations. Econometrica 50 (6), 1345–1370.
MaCurdy, T.E., 1981. An empirical model of labor supply in a life-cycle setting. Journal of Political Economy 89 (6), 1059–1085.
McGrattan, E.R., Rogerson, R., 2004. Changes in hours worked since 1950–2000. Federal Reserve Bank of Minneapolis Quarterly Review 28 (1), 14–33.
Nishiyama, S., Smetters, K., 2007. Does social security privatization produce efficiency gains? Quarterly Journal of Economics 122 (4), 1677–1719.
Prescott, E.C., 2004. Why do Americans work so much more than Europeans? Federal Reserve Bank of Minneapolis Quarterly Review 28 (1), 2–15.
Rogerson, R. and Wallenius, J., (forthcoming). Micro and macro elasticities in a life cycle model with taxes. Journal of Economic Theory.