Supplemental document for "Labor-dependent capital income taxation"

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This document provides additional analyses that are not contained in the paper. In the first part, we discuss in detail how the results of the paper are related to the theory of optimal taxation in a life-cycle model, building on the work of Atkeson, Chari, and Kehoe (1999), Erosa and Gervais (2002) and Garriga (2003). We also discuss how we build on the quantitative study of Conesa, Kitao, and Krueger (2009) on optimal capital taxation in a life-cycle model and "value added" of the paper that introduces labor-dependence in the capital tax function. The second part of this document provides additional numerical analysis that is not included in the paper due to a space constraint.

1 Theory of optimal taxation in a life-cycle model and intuition with simple models

In this section we study the optimal taxation of income using simple models to highlight the intuition of our quantitative results in the full-blown life-cycle model of the paper. We build on the work of Atkeson et al. (1999), Erosa and Gervais (2002) and Garriga (2003) that study the optimal age-dependent income taxation in a life-cycle model. The purpose of this section is to highlight the intuition and demonstrate the role of labor-dependent capital taxation that approximates the role of age-dependent labor and capital taxation using simple two and three-period models with analytical solutions.[1]

We will first review the optimal labor and capital taxation in a two-period model, in which labor taxes can be conditioned on age. We will then study a three-period model in which both labor and capital taxes are allowed to be age-dependent.

The two-period model demonstrates that the optimal age-dependent labor tax

[1]Note that the results about the optimal age-dependent taxation in a life-cycle model presented in this section are not original and more general results including theoretical proofs are contained in Atkeson et al. (1999), Erosa and Gervais (2002) and Garriga (2003). Conesa et al. (2009) also present analytical examples using a two-period model, in order to highlight the role of a positive capital tax that mimics the role of age-dependent labor taxation. We reproduce some of the results contained in Conesa et al. (2009) as well in order to highlight the role of labor-dependent taxation introduced in the paper that mimics age-dependent labor taxation in the same way as a flat positive capital tax does and add a three-period model in order to emphasize its role to approximate age-dependent capital taxation.

declines when labor supply falls.[2] We show that a non-zero capital tax can mimic the role of age-dependent labor taxes when the government is not allowed to condition labor tax rates on age. In a model with more than two periods, more than one intertemporal wedge is needed in order to affect the allocation of labor supply. Therefore a constant capital tax can mimic the wedge, but only imperfectly. Note that the role of a non-zero capital tax to approximate the wedge generated by age-dependent labor taxation is independent of preference specification.

When labor taxes can be conditioned on age, the capital tax is zero under the preference that is separable in consumption and leisure. Under the non-separable preference of the form considered in the paper, the optimal capital tax rate is in general non-zero even if labor taxes can be conditioned on age and the capital tax rate increases over the life-cycle in a typical model with a hump-shaped profile of labor supply.

We show that when the government is not allowed to condition the tax rates on age, labor-dependent capital taxation with a negative cross-partial can help generate the wedges created by the age-dependent labor and capital taxes. First, the benefit of work is not simply the wage net of a labor tax, but also the reduction in the capital tax and a higher after-tax return from assets. Since households' assets increase over the life-cycle, the tax system generates a profile of age-dependent effective wage rates that increase in age, causing the same effect as the labor taxes that decline in age. Second, the capital tax schedule is a decreasing function of labor supply. When work hours start to fall after the peak, the capital tax will begin to rise, mimicking the shape of the optimal age-dependent capital taxes.[3] Note that this second effect will not offset the first effect on the intertemporal margin of labor supply, but rather strengthens it, since rising capital taxes imply an increasing opportunity cost of leisure and induce even more work effort as households age.

We use simple two-period and three-period models in section 1.1 and 1.2 to derive some analytical results and intuition of the results. Section 1.3 summarizes the finding from different models and the role of labor-dependence in capital taxation.

1.1 Two-period model

In what follows, we consider a stationary economy in which prices are constant. The taxes are also time-invariant, but allowed to be conditioned on age. Households live for two periods, consume and supply labor in each period. Preferences are given by

$$U(c_{1,t}, l_{1,t}) + \beta U(c_{2,t+1}, l_{2,t+1}) \tag{1}$$

[2] This is a general result under the separable preference, but needs an additional parametric assumption under the non-separable preference as we discuss below.

[3] In a full-blown life-cycle model with a typical hump-shaped productivity and hours profile, the age-dependent capital tax implies a negative capital tax during the initial years of life-cycle when the labor supply continues to rise. The labor-dependent capital tax we consider does not capture this part and the very young households with low labor income will face a positive and high capital tax. The welfare cost, however, will be relatively small quantitatively since the saving is low in the initial years of life-cycle.

subject to the following budget constraints.

$$c_{1,t} + s_t = (1 - \tau_{l_1})\varepsilon_1 l_{1,t} \quad (2)$$
$$c_{2,t+1} = (1 - \tau_{l_2})\varepsilon_2 l_{2,t+1} + [1 + r(1 - \tau_k)]s_t \quad (3)$$

where ε_j denotes the labor productivity of a household at age j, τ_k is a capital tax and τ_{l_j} represents a tax on labor income of age-j households. First order conditions with respect to labor supply and saving are given as

$$\frac{U_{l_{1,t}}}{U_{c_{1,t}}} = -(1 - \tau_{l_1})\varepsilon_1 \quad (4)$$
$$\frac{U_{l_{2,t+1}}}{U_{c_{2,t+1}}} = -(1 - \tau_{l_2})\varepsilon_2 \quad (5)$$
$$\frac{U_{c_{1,t}}}{U_{c_{2,t+1}}} = \beta[1 + r(1 - \tau_k)] = \frac{U_{l_{1,t}}}{U_{l_{2,t+1}}}\frac{(1 - \tau_{l_2})\varepsilon_2}{(1 - \tau_{l_1})\varepsilon_1} \quad (6)$$

The resource constraint of the economy is given as

$$c_{1,t} + c_{2,t} + G + K_{t+1} = F(K_t, L_t) + (1 - \delta)K_t \quad (7)$$

where K_t and $L_t = \varepsilon_1 l_{1,t} + \varepsilon_2 l_{2,t}$ denote per-capita capital and labor.

Following Atkeson et al. (1999), Erosa and Gervais (2002) and Garriga (2003), we use the primal approach to study the optimal taxation and solve the problem of the government that directly chooses the allocations subject to the first order conditions of the households.

The government chooses $\{c_{1,t}, l_{1,t}, c_{2,t+1}, l_{2,t+1}, K_{t+1}\}$ to maximize

$$U(c_{2,0}, l_{2,0})/\gamma + \sum_{t=0}^{\infty} \gamma^t[U(c_{1,t}, l_{1,t}) + \beta U(c_{2,t+1}, l_{2,t+1})]$$

subject to the resource constraint (7) and the implementability constraint

$$U_{c_{1,t}} c_{1,t} + \beta U_{c_{2,t+1}} c_{2,t+1} + U_{l_{1,t}} l_{1,t} + \beta U_{l_{2,t+1}} l_{2,t+1} = 0, \quad (8)$$

which is obtained by substituting prices and taxes in the intertemporal budget constraint using the first order conditions (4)-(6) of a household problem.

Attach Lagrange multiplier $\gamma^t \mu_t$ to the resource constraint (7) and $\gamma^t \lambda_t$ to the implementability constraint (8). In order to obtain analytical characterization of the optimal taxation and derive an intuition, we consider non-separable and separable preference in consumption and leisure.

Non-separable preference: We consider the Cobb-Douglas non-separable preference in consumption and leisure.

$$u(c, l) = \frac{[c^\nu (1 - l)^{1-\nu}]^{1-\sigma}}{1 - \sigma} \quad (9)$$

First order conditions of the Ramsey problem with respect to consumption and capital are given as

$$\mu_t = \frac{\nu}{c_{1,t}}\left[c_{1,t}^{\nu}(1-l_{1,t})^{1-\nu}\right]^{1-\sigma}\left(1+\lambda_t(1-\sigma)\left[\nu-(1-\nu)\frac{l_{1,t}}{1-l_{1,t}}\right]\right) \quad (10)$$

$$\gamma\mu_{t+1} = \frac{\beta\nu}{c_{2,t+1}}\left[c_{2,t+1}^{\nu}(1-l_{2,t+1})^{1-\nu}\right]^{1-\sigma}\left(1+\lambda_t(1-\sigma)\left[\nu-(1-\nu)\frac{l_{2,t+1}}{1-l_{2,t+1}}\right]\right) \quad (11)$$

$$\mu_t = \gamma\mu_{t+1}(1+r) \quad (12)$$

Combining (10)-(12),

$$\beta(1+r) = \frac{\frac{\left[c_{1,t}^{\nu}(1-l_{1,t})^{1-\nu}\right]^{1-\sigma}}{c_{1,t}}}{\frac{\left[c_{2,t+1}^{\nu}(1-l_{2,t+1})^{1-\nu}\right]^{1-\sigma}}{c_{2,t+1}}} * \frac{1+\lambda_t(1-\sigma)\left[\nu-\frac{(1-\nu)l_{1,t}}{1-l_{1,t}}\right]}{1+\lambda_t(1-\sigma)\left[\nu-\frac{(1-\nu)l_{2,t+1}}{1-l_{2,t+1}}\right]}$$

From the first order condition of a household (6),

$$\beta(1+r(1-\tau_k)) = \frac{\frac{\left[c_{1,t}^{\nu}(1-l_{1,t})^{1-\nu}\right]^{1-\sigma}}{c_{1,t}}}{\frac{\left[c_{2,t+1}^{\nu}(1-l_{2,t+1})^{1-\nu}\right]^{1-\sigma}}{c_{2,t+1}}}$$

Combining the two equations, we have

$$\frac{1+r(1-\tau_k)}{1+r} = \frac{1+\lambda_t(1-\sigma)\left[\nu-\frac{(1-\nu)l_{2,t+1}}{1-l_{2,t+1}}\right]}{1+\lambda_t(1-\sigma)\left[\nu-\frac{(1-\nu)l_{1,t}}{1-l_{1,t}}\right]} \quad (13)$$

Capital tax is not zero in general, unless the labor supply is constant and $l_{1,t} = l_{2,t+1}$. If $\sigma > 1$ as we assume in the paper, τ_k is positive if labor supply declines over the two periods, that is, $l_{1,t} > l_{2,t+1}$.[4]

The first order conditions with respect to labor supply in two periods are given as

$$\mu_t = -\frac{1}{\varepsilon_1}\left[1+\lambda_t(1-\sigma)(\nu-\frac{(1-\nu)l_{1,t}}{1-l_{1,t}})+\frac{\lambda_t}{1-l_{1,t}}\right]U_{l_{1,t}} \quad (14)$$

$$\gamma\mu_{t+1} = -\frac{\beta}{\varepsilon_2}\left[1+\lambda_t(1-\sigma)(\nu-\frac{(1-\nu)l_{2,t+1}}{1-l_{2,t+1}})+\frac{\lambda_t}{1-l_{2,t+1}}\right]U_{l_{2,t+1}} \quad (15)$$

Combining these equations with (10) and (11),

$$\frac{\frac{\left[c_{1,t}^{\nu}(1-l_{1,t})^{1-\nu}\right]^{1-\sigma}}{c_{1,t}}}{\frac{\left[c_{2,t+1}^{\nu}(1-l_{2,t+1})^{1-\nu}\right]^{1-\sigma}}{c_{2,t+1}}} * \frac{1+\lambda_t(1-\sigma)\left[\nu-\frac{(1-\nu)l_{1,t}}{1-l_{1,t}}\right]}{1+\lambda_t(1-\sigma)\left[\nu-\frac{(1-\nu)l_{2,t+1}}{1-l_{2,t+1}}\right]} = \frac{\varepsilon_2}{\beta\varepsilon_1}\frac{1+\lambda_t(1-\sigma)(\nu-\frac{(1-\nu)l_{1,t}}{1-l_{1,t}})+\frac{\lambda_t}{1-l_{1,t}}}{1+\lambda_t(1-\sigma)(\nu-\frac{(1-\nu)l_{2,t+1}}{1-l_{2,t+1}})+\frac{\lambda_t}{1-l_{2,t+1}}}\frac{U_{l_{1,t}}}{U_{l_{2,t+1}}}$$

[4]Note that the denominator and numerator in (13) are positive from (10) and (11). In the paper, we assume $\sigma = 4$, which with the calibration of the consumption weight ν implies constant relative risk aversion at approximately 2.1.

Using the first order conditions of households (4) and (5), we obtain

$$\frac{1-\tau_{l_1}}{1-\tau_{l_2}} = \frac{1+\frac{\lambda_t}{1+\lambda_t(1-\sigma)\gamma-(1+\lambda_t(1-\sigma))l_{2,t+1}}}{1+\frac{\lambda_t}{1+\lambda_t(1-\sigma)\gamma-(1+\lambda_t(1-\sigma))l_{1,t}}} \tag{16}$$

As long as $1+\lambda_t(1-\sigma)>0$, the labor tax declines when the labor supply falls.[5]

In summary, with non-separable preference, when labor supply is not constant across ages, not only does the labor tax vary by age but also the capital tax is non-zero.

Separable preference: Consider the following preference that is separable in consumption and leisure.

$$\frac{c^{1-\sigma_1}}{1-\sigma_1}+\chi\frac{(1-l)^{1-\sigma_2}}{1-\sigma_2}. \tag{17}$$

First order conditions with respect to consumption and capital are given as

$$\mu_t = c_{1,t}^{-\sigma_1}[1+\lambda_t(1-\sigma_1)] \tag{18}$$

$$\gamma\mu_{t+1} = \beta c_{2,t+1}^{-\sigma_1}[1+\lambda_t(1-\sigma_1)] \tag{19}$$

$$\mu_t = \gamma\mu_{t+1}(1+r) \tag{20}$$

Combining equations (18)-(20),

$$\left(\frac{c_{1,t}}{c_{2,t+1}}\right)^{-\sigma_1} = \beta(1+r). \tag{21}$$

The Euler equation of a household problem (6) reads as

$$\left(\frac{c_{1,t}}{c_{2,t+1}}\right)^{-\sigma_1} = \beta[1+r(1-\tau_k)] \tag{22}$$

From (21) and (22), the optimal capital tax is zero.

The first order conditions with respect to the labor supply in two periods are given as

$$\mu_t = \frac{1}{\varepsilon_1}\chi(1-l_{1,t})^{-\sigma_2}\left[1+\lambda_t\left(1+\frac{l_{1,t}\sigma_2}{1-l_{1,t}}\right)\right] \tag{23}$$

$$\theta\mu_{t+1} = \frac{\beta}{\varepsilon_2}\chi(1-l_{2,t+1})^{-\sigma_2}\left[1+\lambda_t\left(1+\frac{l_{2,t+1}\sigma_2}{1-l_{2,t+1}}\right)\right] \tag{24}$$

Combining (23) and (24) and using first order conditions of households' problem (4)-(6), we obtain

$$\frac{1-\tau_{l_2}}{1-\tau_{l_1}} = \frac{1+\lambda_t\left(1+\sigma_2\frac{l_{1,t}}{1-l_{1,t}}\right)}{1+\lambda_t\left(1+\sigma_2\frac{l_{2,t+1}}{1-l_{2,t+1}}\right)} \tag{25}$$

The equation implies that the tax rate is higher when more labor is supplied, that is, $\tau_{l_1}>\tau_{l_2}$ when $l_1>l_2$.

[5] Note that the term $[1+\lambda_t(1-\sigma)\gamma-(1+\lambda_t(1-\sigma))l_{1,t}]$ and $[1+\lambda_t(1-\sigma)\gamma-(1+\lambda_t(1-\sigma))l_{2,t+1}]$ on RHS of (16) are positive.

Role of capital taxation: more intuition with general preference: Using the households' optimality conditions (4), (5) and (6), the intertemporal Euler equation for labor supply reads as

$$\frac{\varepsilon_2 U_{l_{1,t}}}{\varepsilon_1 U_{l_{2,t+1}}} = \beta[1 + r(1-\tau_k)]\frac{1-\tau_{l_1}}{1-\tau_{l_2}} \tag{26}$$

The government can create any tax wedge given by age-dependent labor taxes by a non-zero capital tax. The positive capital tax replicates the labor tax that falls in age, $\tau_{l_2} < \tau_{l_1}$. This result is independent of the preference specification, even under the separable preference in which the optimal capital tax is zero when age-dependent labor taxes are available.[6] The optimal intertemporal wedge with respect to labor supply can be generated either by age-dependent labor taxes or a non-zero, proportional capital tax in the absence of age-dependent labor taxes.

Two-period model with labor-dependent capital taxation (problem of households): Consider the form of labor-dependent capital taxation we considered in the paper, that is, $\tau_k(y_L)$, where the capital tax rate declines in labor income, $\tau_k'(y_L) < 0$. Households maximize the preference (1) subject to the budget constraints

$$c_{1,t} + s_t = (1-\tau_{l_1})\varepsilon_1 l_{1,t} \tag{27}$$

$$c_{2,t+1} = (1-\tau_{l_2})\varepsilon_2 l_{2,t+1} + [1 + r(1-\tau_k(\varepsilon_2 l_{2,t+1}))]s_t \tag{28}$$

First order conditions with respect to labor supply and saving are given as

$$\frac{U_{l_{1,t}}}{U_{c_{1,t}}} = -(1-\tau_{l_1})\varepsilon_1 \tag{29}$$

$$\frac{U_{l_{2,t+1}}}{U_{c_{2,t+1}}} = -(1-\tau_{l_2})\varepsilon_2 + r\varepsilon_2\tau_k'(\varepsilon_2 l_{2,t+1})s_t \tag{30}$$

$$\frac{U_{c_{1,t}}}{U_{c_{2,t+1}}} = \beta[1 + r(1-\tau_k(\varepsilon_2 l_{2,t+1}))] \tag{31}$$

Compare the equations (29)-(31) with (4)-(6), the corresponding set of first order conditions in the economy with age-dependent labor taxes and a flat capital tax. Any allocations that the Ramsey planner chooses subject (29)-(31) can be delivered without the labor-dependence of the capital tax by appropriately adjusting the tax rates. The effect of the extra term $(+r\varepsilon_2\tau_k'(\varepsilon_2 l_{2,t+1})s_t) < 0$ in (30) can be 'undone' by appropriately adjusting τ_{l_2} to achieve the desired intratemporal wedge of consumption and labor in the second period. The Ramsey planner's optimal choice of τ_k in (6) can generate any desired intertemporal wedge of consumption and saving in the same way as the labor-dependent capital taxation $\tau_k(\varepsilon_2 l_{2,t+1})$ in (31). In other words, the three instruments $\{\tau_{l_1}, \tau_{l_2}, \tau_k\}$ are just enough to generate the three desired wedges and to implement the desired allocation of labor supply in two periods $\{l_{1,t}, l_{2,t+1}\}$ and saving s_t. An additional degree of freedom given by $\tau_k' \neq 0$ does not provide any additional value.

More generally, in a model of J periods, if the government can condition tax rates on age, it has enough instruments (J labor taxes and $J-1$ capital taxes for each age),

[6] The result is shown more formally in Appendix A.

in order to affect J intratemporal (labor-consumption) wedges and $J-1$ intertemporal (saving-consumption) wedges and labor-dependence of capital tax will be redundant. If, however, the tax rates cannot be age-dependent, there are roles and additional values from labor-dependent capital taxation since it can mimic the role of age-dependent taxation as we discuss below.

Combining the first order conditions (29)-(31), the intertemporal Euler equation of labor supply is given as

$$\frac{\varepsilon_2 U_{l_{1,t}}}{\varepsilon_1 U_{l_{2,t+1}}} = \beta[1+r(1-\tau_k(\varepsilon_2 l_{2,t+1}))]\frac{(1-\tau_{l_1})}{(1-\tau_{l_2})-r\tau_k'(\varepsilon_2 l_{2,t+1})s_t} \tag{32}$$

Compare (32) with the corresponding equation (26), when the capital tax is a constant. When taxes cannot be conditioned on age, a non-zero capital tax can mimic the role of age-dependent labor taxes in both cases. In particular, a positive capital tax can mimic the role of labor taxes that fall in age when labor supply declines. Equation (32) shows that another possibility to generate such an intertemporal wedge with respect to labor supply is to have a negative dependence of capital taxes on labor supply, since the last term $1/(1-r\tau_k'(\varepsilon_2 l_{2,t+1})s_t) < 1$ in the same way as $(1-\tau_{l_1})/(1-\tau_{l_2}) < 1$.

Intuitively, a positive proportional capital tax can mimic the role of labor taxes that decline in age, since a lower after-tax return from saving will make future consumption more expensive, therefore raising the opportunity cost of leisure time. Similar age-dependent work incentives can be generated by the capital tax that declines in labor income. In a typical life-cycle model calibrated to the micro data, households continue to accumulate wealth until they are close to the retirement age. Therefore old-age households have a stronger incentive to extend more work effort since doing so will increase the after-tax return on their large accumulated wealth. In the next section, we show in a three-period model that the wedge generated by the labor-dependent capital tax varies by age and can also mimic the wedges generated by age-dependent labor taxes that may change over the life-cycle.

1.2 Three-period model

In this section we study a three-period model and show that the optimality of zero capital tax remains when the labor taxes can be conditioned on age under the separable preference of the form (17). In the absence of age-dependent labor taxes, a proportional capital tax can mimic the intertemporal tax wedge generated by age-dependent labor taxes but only imperfectly. We show under a general preference specification that capital taxes that vary with labor income can improve allocations since they create an intertemporal wedge that varies by age.

Assume that households live for three periods and choose a pair of consumption and labor supply at each age. Both labor and capital taxes are conditioned on age of households.

Preferences are given by

$$U(c_{1,t}, l_{1,t}) + \beta U(c_{2,t+1}, l_{2,t+1}) + \beta^2 U(c_{3,t+2}, l_{3,t+2}) \tag{33}$$

subject to the following budget constraints.

$$
\begin{aligned}
c_{1,t} + s_{1,t} &= (1-\tau_{l_1})\varepsilon_1 l_{1,t} && (34)\\
c_{2,t+1} + s_{2,t+1} &= (1-\tau_{l_2})\varepsilon_2 l_{2,t+1} + [1+r(1-\tau_{k_2})]s_{1,t} && (35)\\
c_{3,t+2} &= (1-\tau_{l_3})\varepsilon_3 l_{3,t+2} + [1+r(1-\tau_{k_3})]s_{2,t+1} && (36)\\
& && (37)
\end{aligned}
$$

First order conditions with respect to labor supply and saving are given as

$$
\begin{aligned}
\frac{U_{l_{1,t}}}{U_{c_{1,t}}} &= -(1-\tau_{l_1})\varepsilon_1 && (38)\\
\frac{U_{l_{2,t+1}}}{U_{c_{2,t+1}}} &= -(1-\tau_{l_2})\varepsilon_2 && (39)\\
\frac{U_{l_{3,t+2}}}{U_{c_{3,t+2}}} &= -(1-\tau_{l_3})\varepsilon_3 && (40)\\
\frac{U_{c_{1,t}}}{U_{c_{2,t+1}}} &= \beta[1+r(1-\tau_{k_2})] = \frac{U_{l_{1,t}}}{U_{l_{2,t+1}}}\frac{(1-\tau_{l_2})\varepsilon_2}{(1-\tau_{l_1})\varepsilon_1} && (41)\\
\frac{U_{c_{2,t+1}}}{U_{c_{3,t+2}}} &= \beta[1+r(1-\tau_{k_3})] = \frac{U_{l_{2,t+1}}}{U_{l_{3,t+2}}}\frac{(1-\tau_{l_3})\varepsilon_3}{(1-\tau_{l_2})\varepsilon_2} && (42)
\end{aligned}
$$

As before, we use the primal approach to consider the problem of the government that directly chooses the allocation $\{c_{1,t}, l_{1,t}, c_{2,t+1}, l_{2,t+1}, c_{3,t+2}, l_{3,t+2}, K_{t+1}\}$ to maximize

$$
\frac{U(c_{3,0}, l_{3,0})}{\gamma^2} + \frac{U(c_{2,0}, l_{2,0}) + \beta U(c_{3,1}, l_{3,1})}{\gamma} + \sum_{t=0}^{\infty} \gamma^t [U(c_{1,t}, l_{1,t}) + \beta U(c_{2,t+1}, l_{2,t+1}) + \beta^2 U(c_{3,t+2}, l_{3,t+2})]
$$

subject to the resource constraint

$$
c_{1,t} + c_{2,t} + c_{3,t} + G + K_{t+1} = F(K_t, L_t) + (1-\delta)K_t \tag{43}
$$

where per-capita labor is $L_t = \varepsilon_1 l_{1,t} + \varepsilon_2 l_{2,t} + \varepsilon_3 l_{3,t}$. The implementability constraint is given as

$$
U_{c_{1,t}} c_{1,t} + \beta U_{c_{2,t+1}} c_{2,t+1} + \beta^2 U_{c_{3,t+2}} c_{3,t+2} + U_{l_{1,t}} l_{1,t} + \beta U_{l_{2,t+1}} l_{2,t+1} + \beta^2 U_{l_{3,t+2}} l_{3,t+2} = 0 \tag{44}
$$

Attach Lagrange multiplier $\gamma^t \mu_t$ to the resource constraint (43) and $\gamma^t \lambda_t$ to the implementability constraint (44).

Non-separable preference: Consider the non-separable preference of the form (9). Following the same steps as in the two-period model, we obtain the condition corresponding to (13) in the two-period model.

$$
\frac{1 + r(1-\tau_{k_{j+1}})}{1+r} = \frac{1 + \lambda_t(1-\sigma)\left[\nu - \frac{(1-\nu)l_{j+1,t+1}}{1-l_{j+1,t+1}}\right]}{1 + \lambda_t(1-\sigma)\left[\nu - \frac{(1-\nu)l_{j,t}}{1-l_{j,t}}\right]} \tag{45}
$$

for $j = 1, 2$. The condition can be generalized for a model with any finite maximum age J. It implies that the optimal capital tax is positive (negative) when labor supply falls (increases) and the absolute value of the tax will be larger when the labor supply increases or decreases faster. In a typical life-cycle model in which productivity and labor supply exhibit a hump-shape with a single peak, the optimal capital tax will be negative in the initial years and changes the sign at the peak of the labor supply. Thereafter, the capital tax remains positive and rises as households age and labor supply declines. These results are shown by Erosa and Gervais (2002) and also by Garriga (2003). Figure 1 is from Erosa and Gervais (2002), in which they compute the optimal profile of capital and labor taxes under the a Cobb-Douglas preference in consumption and leisure.

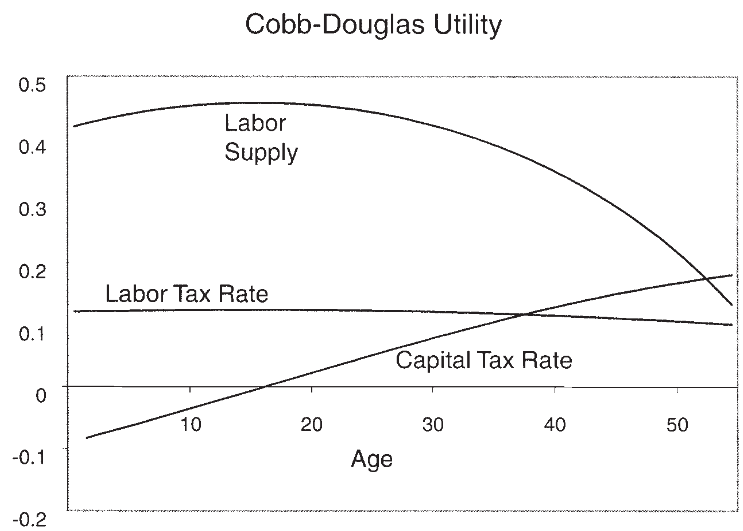


FIG. 3. Labor supply and tax rates over the lifetime of individuals.

Figure 1: Optimal age-dependent taxation from Erosa and Gervais (2002)

From the first order conditions with respect to labor supply, we obtain the condition similar to (16)

$$\frac{1-\tau_{l_j}}{1-\tau_{l_{j+1}}} = \frac{1+\frac{\lambda_t}{1+\lambda_t(1-\sigma)\gamma-(1+\lambda_t(1-\sigma))l_{j+1,t+1}}}{1+\frac{\lambda_t}{1+\lambda_t(1-\sigma)\gamma-(1+\lambda_t(1-\sigma))l_{j,t}}} \tag{46}$$

for $j = 1, 2$. We have $\tau_{l_j} > \tau_{l_{j+1}}$ when $l_j > l_{j+1}$, as long as $1 + \lambda_t(1-\sigma) > 0$.

Separable preference: Consider the separable preference of the form (17). First order conditions for consumption and capital are given as

$$c_{1,t}^{-\sigma_1} = \mu_t - \lambda_t(1-\sigma_1)c_{1,t}^{-\sigma_1} \tag{47}$$

$$\beta c_{2,t+1}^{-\sigma_1} = \gamma\mu_{t+1} - \lambda_t\beta(1-\sigma_1)c_{2,t+1}^{-\sigma_1} \tag{48}$$

$$\beta^2 c_{3,t+2}^{-\sigma_1} = \gamma^2\mu_{t+2} - \lambda_t\beta^2(1-\sigma_1)c_{3,t+2}^{-\sigma_1} \tag{49}$$

$$\mu_t = \gamma\mu_{t+1}(1+r) \tag{50}$$

Combining the equations,

$$\left(\frac{c_{1,t}}{c_{2,t+1}}\right)^{-\sigma} = \left(\frac{c_{2,t+1}}{c_{3,t+2}}\right)^{-\sigma} = \beta(1+r) \tag{51}$$

Comparing with the Euler equations of a household problem (41) and (42) under the separable preference,

$$\left(\frac{c_{1,t}}{c_{2,t+1}}\right)^{-\sigma_1} = \beta[1 + r(1-\tau_{k_2})] \tag{52}$$

$$\left(\frac{c_{2,t+1}}{c_{3,t+2}}\right)^{-\sigma_1} = \beta[1 + r(1-\tau_{k_3})] \tag{53}$$

the optimal capital tax $\tau_{k_2} = \tau_{k_3} = 0$. As Proposition 3.3 of Erosa and Gervais (2002) shows, this result extends to a model with any finite maximum age of a household.

First order conditions with respect to the labor supply are given as

$$\mu_t \varepsilon_1 = \chi(1-l_{1,t})^{-\sigma_2}[1 + \lambda_t(1 + \sigma_2 \frac{l_{1,t}}{1-l_{1,t}})] \tag{54}$$

$$\gamma \mu_{t+1} \varepsilon_2 = \beta\chi(1-l_{2,t+1})^{-\sigma_2}[1 + \lambda_t(1 + \sigma_2 \frac{l_{2,t+1}}{1-l_{2,t+1}})] \tag{55}$$

$$\gamma^2 \mu_{t+2} \varepsilon_3 = \beta^2\chi(1-l_{3,t+2})^{-\sigma_2}[1 + \lambda_t(1 + \sigma_2 \frac{l_{3,t+2}}{1-l_{3,t+2}})] \tag{56}$$

Using households' first order conditions (41) and (42),

$$\frac{1-\tau_{l_2}}{1-\tau_{l_1}} = \frac{1 + \lambda_t(1 + \sigma_2 \frac{l_{1,t}}{1-l_{1,t}})}{1 + \lambda_t(1 + \sigma_2 \frac{l_{2,t+1}}{1-l_{2,t+1}})} \tag{57}$$

$$\frac{1-\tau_{l_3}}{1-\tau_{l_2}} = \frac{1 + \lambda_t(1 + \sigma_2 \frac{l_{2,t+1}}{1-l_{2,t+1}})}{1 + \lambda_t(1 + \sigma_2 \frac{l_{3,t+2}}{1-l_{3,t+2}})} \tag{58}$$

In the steady state, the conditions imply $\tau_{l_j} > \tau_{l_{j+1}}$ if and only if $l_j > l_{j+1}$. The government optimally chooses to condition labor taxes on age when the labor supply profile is not completely flat across ages and taxes labor supply of households more heavily when they work more.

Role of capital taxation: more intuition with general preference: Using the first order conditions (38)-(42), the intertemporal conditions for labor supply are given as

$$\frac{\varepsilon_2 U_{l_{1,t}}}{\varepsilon_1 U_{l_{2,t+1}}} = \beta[1 + r(1-\tau_{k_2})]\frac{1-\tau_{l_1}}{1-\tau_{l_2}} \tag{59}$$

$$\frac{\varepsilon_3 U_{l_{2,t+1}}}{\varepsilon_2 U_{l_{3,t+2}}} = \beta[1 + r(1-\tau_{k_3})]\frac{1-\tau_{l_2}}{1-\tau_{l_3}} \tag{60}$$

As in the two-period model, the wedge created by age-dependent labor tax can be mimicked by capital taxation that depends on age. If, however, labor and capital taxes are both restricted to be age-independent, the optimal wedge cannot be perfectly mimicked since a constant capital tax rate is not able to generate two different wedges $(1-\tau_{l_1})/(1-\tau_{l_2})$ and $(1-\tau_{l_2})/(1-\tau_{l_3})$. In this case, we show below that there is a potential for labor-dependent capital taxation to improve the allocations.

Note that non-zero proportional capital tax and labor-dependent capital tax are not the only ways to approximate the optimal age-dependent labor taxation. In a related paper, Gervais (2009) studies the effect of progressive labor income taxation in the U.S. and argues that it (imperfectly) mimics the optimal profile of age-dependent labor taxes. He shows that a combination of progressive labor taxes and a relatively high capital tax can improve the welfare relative to the optimal flat taxes on capital and labor. In terms of the intertemporal conditions with respect to labor supply (59) and (60), when labor income declines (increases), the marginal tax rates will fall (increase) due to the progressivity and creates similar age-varying wedges as those generated by age-dependent labor taxes.[7]

Three-period model with labor-dependent capital taxation (problem of households): We consider the labor-dependent capital taxation $\tau_k(y_L)$, which declines in labor income. Households maximize the preference (33) subject to the budget constraints

$$c_{1,t} + s_{1,t} = (1-\tau_{l_1})\varepsilon_1 l_{1,t} \tag{61}$$

$$c_{2,t+1} + s_{2,t+1} = (1-\tau_{l_2})\varepsilon_2 l_{2,t+1} + [1 + r(1-\tau_k(\varepsilon_2 l_{2,t+1})]s_{1,t} \tag{62}$$

$$c_{3,t+2} = (1-\tau_{l_3})\varepsilon_3 l_{3,t+2} + [1 + r(1-\tau_k(\varepsilon_3 l_{3,t+2})]s_{2,t+1} \tag{63}$$

First order conditions with respect to labor supply and saving are given as

$$\frac{U_{l_{1,t}}}{U_{c_{1,t}}} = -(1-\tau_{l_1})\varepsilon_1 \tag{64}$$

$$\frac{U_{l_{2,t+1}}}{U_{c_{2,t+1}}} = -(1-\tau_{l_2})\varepsilon_2 + r\varepsilon_2\tau_k'(\varepsilon_2 l_{2,t+1})s_{1,t} \tag{65}$$

$$\frac{U_{l_{3,t+2}}}{U_{c_{3,t+2}}} = -(1-\tau_{l_3})\varepsilon_3 + r\varepsilon_3\tau_k'(\varepsilon_3 l_{3,t+2})s_{2,t+1} \tag{66}$$

$$\frac{U_{c_{1,t}}}{U_{c_{2,t+1}}} = \beta[1 + r(1-\tau_k(\varepsilon_2 l_{2,t+1}))] \tag{67}$$

$$\frac{U_{c_{2,t+1}}}{U_{c_{3,t+2}}} = \beta[1 + r(1-\tau_k(\varepsilon_3 l_{3,t+2}))] \tag{68}$$

Combining the first order conditions, the intertemporal conditions for labor supply are given as

$$\frac{\varepsilon_2 U_{l_{1,t}}}{\varepsilon_1 U_{l_{2,t+1}}} = \beta[1 + r(1-\tau_k(\varepsilon_2 l_{2,t+1}))]\frac{(1-\tau_{l_1})}{(1-\tau_{l_2}) - r\tau_k'(\varepsilon_2 l_{2,t+1})s_{1,t}}$$

$$\frac{\varepsilon_3 U_{l_{2,t+1}}}{\varepsilon_2 U_{l_{3,t+2}}} = \beta[1 + r(1-\tau_k(\varepsilon_3 l_{3,t+2}))]\frac{(1-\tau_{l_2}) - r\tau_k'(\varepsilon_2 l_{2,t+1})s_{1,t}}{(1-\tau_{l_3}) - r\tau_k'(\varepsilon_3 l_{3,t+2})s_{2,t+1}}$$

[7]More precisely, the wedges will vary exactly in the same direction if the labor income and labor supply are perfectly correlated, but in general, both profiles are hump-shaped and peaks are close to each other.

It is typically the case that in a life-cycle model with a hump-shaped productivity profile the labor supply starts to decline at middle ages and falls more sharply as households approach the retirement age.[8] As we saw in equations (57) and (58) in the case of separable preference, it implies that the optimal wedge $\omega_j = (1-\tau_j)/(1-\tau_{j+1})$ created by the age-dependent labor tax is less than unity and declines as households age. If labor taxes cannot be conditioned on age, a positive capital tax can mimic the role of labor taxes that decline in age, but a constant capital tax can do so only imperfectly as we discussed above. More generally speaking, a single capital tax is unable to replicate $J-1$ wedges (with $J > 2$) needed for achieving the optimal profile of labor supply in a model of households with the maximum working age of J.

In addition, if labor income declines in age, that is, $\varepsilon_2 l_{2,t+1} > \varepsilon_3 l_{3,t+2}$, the after-tax return from saving $r[1-\tau_k(\varepsilon_j l_{j,t})]$ falls in age j as well. This mimics the intertemporal *wedge* ω_j under the age-dependent labor taxation that declines in age and presents the possibility of improving the allocations upon those implied by a constant capital tax.

Note that the term $(-r\tau_k'(\varepsilon_{j+1} l_{j+1,t+1})s_{j,t}) > 0$ on the RHS is increasing in the amount of saving $s_{j,t}$. In the absence of age-dependent labor tax, this extra term can mimic the role of a labor tax that declines in age and can improve upon the allocations implied by a constant labor tax. Allowing for labor-dependence of capital taxation provides more work incentive for older households that have accumulated more wealth.

We also note that the other consequence of using a constant positive capital tax to mimic the role of labor taxes is the general equilibrium effect on aggregate capital and output through the intertemporal distortions on saving decisions. Recall that under the separable preference, the optimal capital tax when the labor tax is allowed to be age-dependent was zero. A positive capital tax that departs from the optimality faces a trade-off between its role of mimicking the age-dependent labor taxation and distorting the intertemporal consumption allocation and discouraging savings. With labor-dependent capital tax, the wedge can be also generated by the negative derivative of capital tax function interacting with the rising saving, $(1-r\tau_k'(\varepsilon_{j+1} l_{j+1,t+1})s_{j,t})/(1-r\tau_k'(\varepsilon_{j+2} l_{j+2,t+2})s_{j+1,t+1})$, and we do not need to rely solely on a high proportional capital tax, mitigating the negative effects of a high capital tax. In addition, additional saving will not only increase future consumption, but also effective wage rates in future since the extra benefit of work due to the reduction in capital taxes increases in wealth.

1.3 Summary

- With age-dependent labor taxes, $\tau_{l_j} > \tau_{l_{j+1}}$ when $l_j > l_{j+1}$ in general under both separable and non-separable preference.

- When taxes can be conditioned on age, the optimal capital tax is zero under the separable preference. Under the non-separable preference, the optimal capital tax is non-zero in general and it is positive when the labor supply is rising and larger in magnitude when the growth rate of labor supply is higher.

- Without age-dependent labor tax, a non-zero capital tax can mimic the wedge gen-

[8] See for example Figure 2 in Erosa and Gervais (2002) or Figure 1 in Conesa et al. (2009).

erated by age-dependent labor taxes in a two-period model, under both separable and non-separable preference specifications.

In a model with more than two periods, the government can use age-dependent capital taxes to mimic intertemporal wedges with respect to labor supply that are implied by the age-dependent labor taxes. They can, however, do so only imperfectly if capital tax rates cannot be conditioned on age either. A constant capital tax can create only one wedge whereas there can be as many as $J - 1$ wedges if the labor supply is never constant over the life-cycle.

Roles of $\tau_k' < 0$:

1. $\tau_k' < 0$ can create a wedge that mimics the role of age-dependent labor taxes, in the same way as a constant capital tax can do.

   Intuitively, the return from additional work effort comes not only from the wage net of a labor tax, but also from a lower capital tax on savings and a higher after-tax return. Such a gain will rise in age when the saving also increases in age, just in the same way as the optimal age-dependent labor tax declines. Essentially, both taxes generate a profile of effective wage rates that increase in age.

2. $\tau_k' < 0$ implies a profile of capital taxes that rise in age in the part of the life-cycle where the labor productivity and labor supply decline, which approximates the profile of optimal age-dependent capital taxes that increase in age after the peak in labor supply.

   A higher capital tax will increase the opportunity cost of leisure, giving more incentive to work as households age in the same way as the age-dependent labor taxes that decline in age.[9]

3. The intertemporal wedge with respect to labor supply generated by age-dependent labor taxes can be approximated by the above two effects under labor-dependent capital taxation. In the case of a constant capital tax, in which a single capital tax has to do the job by itself, the optimal capital tax tends to be significantly high, distorting the intertemporal allocation of consumptions and reducing households' saving and aggregate capital of the economy. The negative equilibrium effect on saving can possibly be mitigated by having the two channels and an intertemporal labor allocation can be improved without a large general equilibrium effect on aggregate capital (and output and consumption).

2 Additional numerical analysis

2.1 Role of price adjustments: partial equilibrium analysis

In order to understand the role of general equilibrium effects of price adjustment, we conduct a reform experiment under the partial equilibrium assumption, holding the interest

[9] This is the force that drives the optimality of a high capital tax in a life-cycle model emphasized in Conesa et al. (2009), in which the capital tax is independent of labor supply.

rate and wage fixed at the benchmark level during the transition and in the final steady state. Table 1 summarizes the results. Since the interest rate is pinned at the lower level of 5.0% than it actually is during the transition and in the final steady state under the general equilibrium, households saving declines and aggregate capital falls by 3.9%. There is not a large change in the steady state welfare, but the transitional welfare gain is higher for the new-born at the time of the reform since they would not suffer from the lower wage rate during the transition.

Table 1: Reform effects under general vs partial equilibrium

	General eq.	Partial eq.
Output Y	+4.8%	+2.3%
Capital K	+2.1%	−3.9%
Labor L	+6.3%	+5.9%
Wage	−1.4%	unch.
Interest rate	+0.3%-pt	unch.
CEV (steady state, ex-ante)	+3.7%	+3.6%
CEV (transition, new-born avg)	+2.4%	+2.9%

2.2 Wealth distribution of the model and changes in inequality

Figure 2 shows the cumulative distribution of wealth in the benchmark economy for workers and retirees. The model generates ex-post heterogeneity among households in earnings and wealth induced by the labor productivity shocks and it generates a fair degree of wealth inequality. The 5% wealthiest households own 24.4% of the entire wealth in the benchmark economy and top 10%, 20% and 40% own 40.5%, 63.1% and 88.1%, respectively. We do not, however, perfectly capture the wealth distribution as in the United States. In particular, the model does not generate the extremely high concentration of wealth at the right end of the distribution, in which top 5% wealthiest hold 50% of the entire wealth in the U.S. (Budría Rodríguez et al, 2002). In order to generate such a high degree of concentration, a model would need be enriched with additional features such as productive entrepreneurs and their earnings dynamics, who constitute more than 50% of the top 5% wealthiest (Cagetti and De Nardi 2008, Quadrini 2000), intergenerational linkages of productivity and bequest motives as in De Nardi (2004), or some exogenous productivity process with highly persistent extraordinarily good productivity shocks as in Castañeda et al. (2003).[10]

With the labor-dependent capital tax reform, the wealth distribution becomes more dispersed, as shown in Figure 3. The incentive to work harder will operate more intensely on those with more wealth, who also tend to have higher productivity, and accelerate their wealth accumulation, increasing the wealth inequality. The Gini coefficient for wealth will rise from 0.62 in the benchmark to 0.72 under the reform.

[10]We do not incorporate such features in order to keep the model tractable and focus on the effect of the tax reform in a way that is comparable to the findings of existing works such as Erosa and Gervais (2002) and Conesa et al. (2009). As we mentioned in the conclusion, we leave it for future research to study the effect of tax reforms in a model that captures more heterogeneity among households.

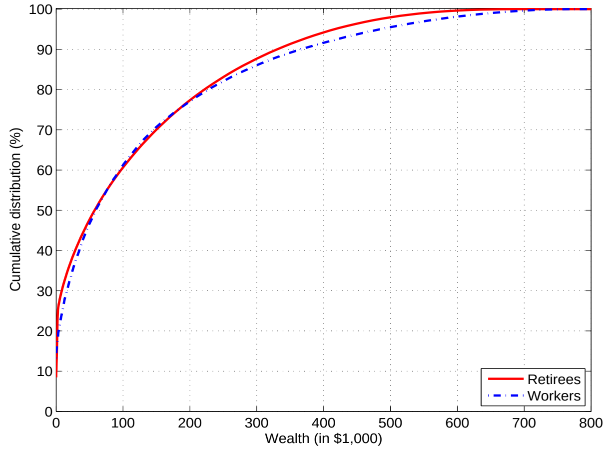
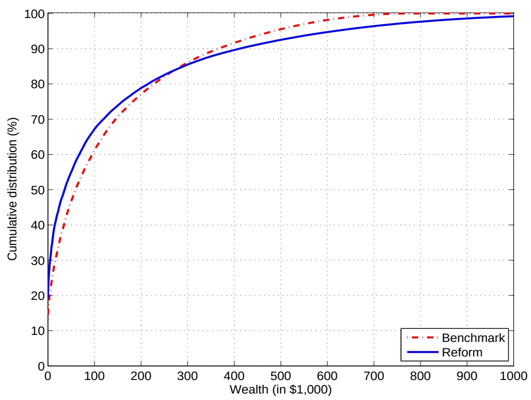
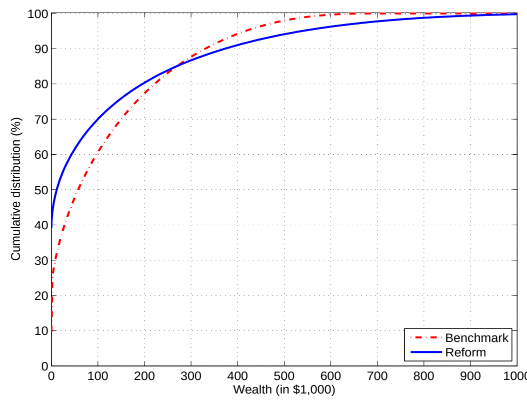


Figure 2: Wealth distribution in the benchmark economy



(a) Workers



(b) Retirees

Figure 3: Wealth distribution in the benchmark and reform economy

References

Atkeson, A., V. Chari, and P. Kehoe (1999). Taxing capital income: A bad idea. *Federal Reserve Bank of Minneapolis, Quarterly Review 23*(3), 3–17.

Budría Rodríguez, S., J. Díaz-Giménez, V. Quadrini, and J.-V. Ríos-Rull (2002). Updated facts on the U.S. distributions of earnings, income, and wealth. *Federal Reserve Bank of Minneapolis, Quarterly Review 26*(3), 2–35.

Cagetti, M. and M. De Nardi (2008). Wealth inequality: Data and models. *Macroeconomic Dynamics 12*(S2), 285313.

Castañeda, A., J. Díaz-Giménez, and J.-V. Ríos-Rull (2003). Accounting for the U.S. earnings and wealth inequality. *Journal of Political Economy 111*(4), 818–857.

Conesa, J. C., S. Kitao, and D. Krueger (2009). Taxing capital? Not a bad idea after all! *American Economic Review 99*(1), 25–48.

De Nardi, M. (2004). Wealth inequality and intergenerational links. *Review of Economic Studies 71*(3), 743–768.

Erosa, A. and M. Gervais (2002). Optimal taxation in life-cycle economies. *Journal of Economic Theory 105*, 338–369.

Garriga, C. (2003). Optimal fiscal policy in overlapping generations models. mimeo, Florida State University.

Gervais, M. (2009). On the optimality of age-dependent taxes and the progressive U.S. tax system. Working Paper, University of Southampton.

Quadrini, V. (2000). Entrepreneurship, saving, and social mobility. *Review of Economic Dynamics 3*, 1–40.

A Capital taxation with age-independent labor taxes: separable preference

This section shows that the optimal capital tax is non-zero under the separable preference if labor taxes cannot be conditioned on age.

Two-period model: Imposing the age-independent labor tax will add the following constraint to the Ramsey problem.

$$U_{l_{1,t}} U_{c_{2,t}} \varepsilon_2 = U_{l_{2,t}} U_{c_{1,t}} \varepsilon_1. \tag{69}$$

Attach Lagrange multiplier $\gamma^t \eta_t$ to (69), the intertemporal condition of consumption (21) becomes

$$\left(\frac{c_{1,t}}{c_{2,t+1}}\right)^{-\sigma_1} = \beta(1+r) \frac{1 + \lambda_t(1-\sigma_1) + \eta_t \chi \frac{\sigma_1 (1-l_{2,t})^{-\sigma_2}}{c_{1,t}}}{1 + \lambda_t(1-\sigma_1) - \eta_{t+1} \chi \frac{\sigma_1 (1-l_{1,t+1})^{-\sigma_2}}{c_{2,t+1}}} \tag{70}$$

Combining with the households' Euler equation (22), we have

$$\frac{1 + r(1-\tau_k)}{1+r} = \frac{1 + \lambda_t(1-\sigma_1) + \eta_t \chi \frac{\sigma_1 (1-l_{2,t})^{-\sigma_2}}{c_{1,t}}}{1 + \lambda_t(1-\sigma_1) - \eta_{t+1} \chi \frac{\sigma_1 (1-l_{1,t+1})^{-\sigma_2}}{c_{2,t+1}}} \tag{71}$$

Therefore τ_k is non-zero as long as the constraint (69) binds. If the government with access to age-dependent labor taxes would like to set $\tau_{l_1} < \tau_{l_2}$, it implies the Lagrange multiplier $\eta < 0$ and $\tau_k > 0$.

Three-period model: When the labor tax is restricted to be age-independent, that is, $\tau_{l_1} = \tau_{l_2} = \tau_{l_3}$, the additional constraints in the Ramsey problem are given as

$$U_{l_{1,t}} U_{c_{2,t}} \varepsilon_2 = U_{l_{2,t}} U_{c_{1,t}} \varepsilon_1 \tag{72}$$
$$U_{l_{2,t}} U_{c_{3,t}} \varepsilon_3 = U_{l_{3,t}} U_{c_{2,t}} \varepsilon_2 \tag{73}$$

Attach Lagrange multipliers $\gamma^t \eta_{1,t}$ and $\gamma^t \eta_{2,t}$ to (72) and (73) respectively. The first order conditions with respect to consumption and capital will become

$$\begin{aligned}
\mu_t &= c_{1,t}^{-\sigma_1}\left[1+\lambda_t(1-\sigma_1)+\eta_{1,t}\frac{\chi(1-l_{2,t})^{-\sigma_2}\sigma_1\varepsilon_1}{c_{1,t}}\right] \\
\gamma\mu_{t+1} &= \beta c_{2,t+1}^{-\sigma_1}\left[1+\lambda_t(1-\sigma_1)-\frac{\chi[\eta_{1,t+1}(1-l_{1,t+1})^{-\sigma_2}-\eta_{2,t+1}(1-l_{3,t+1})^{-\sigma_2}]\sigma_1\varepsilon_2}{c_{2,t+1}}\right] \\
\gamma^2\mu_{t+2} &= \beta^2 c_{3,t+2}^{-\sigma_1}\left[1+\lambda_t(1-\sigma_1)-\eta_{2,t+2}\frac{\chi(1-l_{2,t+2})^{-\sigma_2}\sigma_1\varepsilon_3}{c_{3,t+2}}\right]
\end{aligned}$$

Combining the equations

$$\beta(1+r) = \left(\frac{c_{1,t}}{c_{2,t+1}}\right)^{-\sigma_1}\Omega_1 \tag{74}$$

$$\beta(1+r) = \left(\frac{c_{2,t+1}}{c_{3,t+2}}\right)^{-\sigma_1}\Omega_2 \tag{75}$$

where

$$\begin{aligned}
\Omega_1 &= \frac{1+\lambda_t(1-\sigma_1)+\eta_{1,t}\frac{\chi(1-l_{2,t})^{-\sigma_2}\sigma_1\varepsilon_1}{c_{1,t}}}{1+\lambda_t(1-\sigma_1)-\frac{\chi[\eta_{1,t+1}(1-l_{1,t+1})^{-\sigma_2}-\eta_{2,t+1}(1-l_{3,t+1})^{-\sigma_2}]\sigma_1\varepsilon_2}{c_{2,t+1}}} \\
\Omega_2 &= \frac{1+\lambda_t(1-\sigma_1)-\frac{\chi[\eta_{1,t+1}(1-l_{1,t+1})^{-\sigma_2}-\eta_{2,t+1}(1-l_{3,t+1})^{-\sigma_2}]\sigma_1\varepsilon_2}{c_{2,t+1}}}{1+\lambda_t(1-\sigma_1)-\eta_{2,t+2}\frac{\chi(1-l_{2,t+2})^{-\sigma_2}\sigma_1\varepsilon_3}{c_{3,t+2}}}
\end{aligned}$$

Comparing with the Euler equations of a household

$$\begin{aligned}
\beta[1+r(1-\tau_{k_2})] &= \left(\frac{c_{1,t}}{c_{2,t+1}}\right)^{-\sigma_1} \\
\beta[1+r(1-\tau_{k_3})] &= \left(\frac{c_{2,t+1}}{c_{3,t+2}}\right)^{-\sigma_1}
\end{aligned}$$

τ_{k_2} and τ_{k_3} are non-zero as long as the constraints (72) and (73) bind and $\tau_{k_3} \neq \tau_{k_2}$ in general. Without age-dependent labor taxes, optimal capital tax is non-zero and it does vary by age if it is allowed to do so.